Vibration Analysis of Thin Rotating Cylindrical Shell

A Thesis Submitted In Partial Fulfillment

of the Requirements for the degree of

Master of Technology

In

Civil Engineering

(Structural Engineering)

By

RAKESH SHAMBHARKAR

Department of Civil Engineering

NATIONAL INSTITUTE OF TECHNOLOGY ROURKELA
Rourkela-769008,
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RAKESH SHAMBHARKAR

Roll No-20601027

Under The Guidance of

Prof. A. V. Asha

Department of Civil Engineering

National Institute of Technology Rourkela
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NATIONAL INSTITUTE OF TECHNOLOGY
ROURKELA – 769008, ORISSA
INDIA

CERTIFICATE

This is to certify that the thesis entitled, **"VIBRATION ANALYSIS OF THIN ROTATING CYLINDRICAL SHELL"** submitted by **Mr. Rakesh Shambharkar** in partial fulfillment of the requirement for the award of **Master of Technology** Degree in **Civil Engineering** with specialization in **Structural Engineering** at the National Institute of Technology, Rourkela (Deemed University) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University/ Institute for the award of any degree or diploma.

Date: May 30, 2008

Place: Rourkela

Prof. A. V. Asha
Dept of Civil Engineering
National Institute of Technology
Rourkela – 769008

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Rakesh Shambharkar

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ABSTRACT

With the continually increasing use of turbo machinery at higher performance levels, especially in aircraft, the study of vibration problems arising in rotating blades has become increasingly important. Free vibration frequencies and mode shapes are essential for the analysis of resonant response and flutter. Due to its significance in structural mechanics, many researchers have worked on the vibration characteristics of turbo machinery blades.

Rotating circular shell structures in many engineering applications like aviation, rocketry, missiles, electric motors and locomotive engines are increasingly used. They find increasing application in aerospace, chemical, civil and mechanical industries such as in high-speed centrifugal separators, gas turbines for high-power aircraft engines, spinning satellite structures, certain rotor systems and rotating magnetic shields. In many cases, a rotating shell may be one of the main vibration and noise sources. In order to reduce the vibration, noise and to increase the strength of shells or shafts, it is therefore very important for engineers to understand the vibration of shells and design suitable shells with low vibration and noise radiation characteristics. Thus, frequencies and mode shapes of such structures are important in the design of systems.

Composite structures have extensive use in aerospace, civil, marine and other engineering applications. Laminated composites are becoming key components in many of them. Their high performance places them at the top of the list of engineering materials needed for advanced design applications. This is because controlling the lamination angle can alter their structural properties and the stacking sequence leading to an optimal design. The higher specific modulus and specific strength of these composites means that the weight of certain components can be reduced. The increasingly wider application to other fields of engineering has necessitated the evolution of adequate analytical tools for the better understanding of the structural behavior and efficient utilization of the materials.

In this thesis work, an analytical solution of frequency characteristics for the vibrations of rotating laminated composite cylindrical thin shells by using the “first order shear deformation theory”. Compared with classical theory and higher order theory, the “first order shear deformation theory” combines higher accuracy and lower calculation efforts. The objective of this study is to examine the effect of various shell parameters on the frequency characteristics of rotating laminated composite cross-ply thin shells. For reasons of simplicity, the simply supported boundary conditions at both ends of the shells. Figures show variation of frequency with the rotating speed. The formulation is general. Different boundary conditions, lamination schemes (which may be isotropic or orthotropic), order of shear deformation theories, and even forms of assumed solutions can be easily accommodated into the analysis.

LIST OF SYMBOLS

The principal symbols used in this thesis are presented for easy reference. A symbols is used for different meaning depending on the context and defined in the text as they occur.

English

Notation	**Description**
A_1 , A_2	Lames Parameter or Surface Metrics
A_{ij} , B_{ij} , D_{ij}	Laminate Stiffness's
a_1 , a_2	Element of area of the cross section
C , $\overline{C}$	The Matrices notation described in appendix
dS	The distance between points (a, b, z) and $(a + da, b + db, z + dz)$
dV	Volume of the shell element
dt	Time derivative
E_1 , E_2	Longitudinal and transverse elastic moduli respectively
G_{12} , G_{13} , G_{23}	In plane and Transverse shear moduli
h	Total thickness of the shell
h_k	Distance from the reference surface to the layer interface
k_i	The shear correction factors.
k_1 , k_2 , k_6	Quantities whose expression are given in equation [11.b]
L_1 , L_2 , L_3	Lames Coefficient

L	Length of the cylindrical shell
N_i , M_i , Q_1 , Q_2	Stress resultant and stress couples
M , $\overline{M}$	The Matrices notation described in appendix
m	Axial half wave number
n	Circumferential wave number
$Q_{ij}^{(k)}$	Reduced stiffness matrix of the constituent layer
R	Radius of the reference surface of cylindrical shell
R_1 , R_2	Principal radii of curvature
t	Time
T	Kinetic energy
U , V, W , Φ_1 , Φ_2	Amplitude of displacement
U_s	Strain energy
$\overline{U}$, $\overline{V}$, $\overline{W}$, $\overline{\Phi}_1$, $\overline{\Phi}_2$	Non dimensionalised amplitude of displacement
u , v , w	Displacement component at the reference surface
$\overline{u}$, $\overline{v}$, $\overline{w}$	Non dimensionalised Displacement component at any point in the shell
V_L	Potential of all applied loads
$\{X\}$	Column matrix of amplitude of vibration or eigenvector.
$\{\overline{X}\}$	Non dimensionalised form of column matrix

Greek

Notation	Description
a , b , z	Shell coordinates
d	Mathematical operation called variation
$\frac{\partial \bar{r}}{\partial a}$, $\frac{\partial \bar{r}}{\partial b}$	The vectors are tangent to the a and b coordinate lines
e_i	Strain component
e_1^0 , e_2^0 , e_4^0 , e_5^0 , e_6^0	Quantities whose expression are given in equation [11.b]
l_m	Non dimensionalised axial wave parameter
m_{12}	Major Poisson’s ratio
r	Material density
s_i	Stress Component
f_1 , f_2	Rotation at z = 0 of normal’s to the mid-surface with respect to $a-$and $b-$ axes
w	Natural circular Frequency parameter
$\bar{v}$	Dimensionalised Frequency parameter
Ω	Rotating velocity of circular cylindrical shell

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INTRODUCTION

CHAPTER 1
INTRODUCTION

With the continually increasing use of turbo machinery at higher performance levels, especially in aircraft, the study of vibration problems arising in rotating blades has become increasingly important. Free vibration frequencies and mode shapes are essential for the analysis of resonant response and flutter. Due to its significance in structural mechanics, many researchers have worked on the vibration characteristics of turbo machinery blades.

The turbine blading carefully designed with the correct aerodynamic shape to properly turn the flowing steam and generate rotational energy efficiently. The blades also have to be strong enough to withstand high centrifugal stresses and it size should to be avoid dangerous vibrations. Various types of blading arrangements proposed, but the designed are to take advantage of the principle that when a given mass of steam suddenly changes its velocity, a force exerted by the mass in direct proportion to the rate of change of velocity.

Rotating circular shell structures in many engineering applications like aviation, rocketry, missiles, electric motors and locomotive engines are increasingly used. They find increasing application in aerospace, chemical, civil and mechanical industries such as in high-speed centrifugal separators, gas turbines for high-power aircraft engines, spinning satellite structures, certain rotor systems and rotating magnetic shields. In many cases, a rotating shell may be one of the main vibration and noise sources. In order to reduce the vibration, noise and to increase the strength of shells or shafts, it is therefore very important for engineers to understand the vibration of shells and design suitable shells with low vibration and noise radiation characteristics. Thus, frequencies and mode shapes of such structures are important in the design of systems.

Spinning cylindrical shells in various industrial equipments like gas turbines, locomotive engines, high-speed centrifugal separators and rotor systems are used.

Because of this, the study on the vibration of spinning cylindrical shells is essential to understanding of rotating structures; many researchers have been interested in this topic.

Composite structures have extensive use in aerospace, civil, marine and other engineering applications. Laminated composites are becoming key components in many of them. Their high performance places them at the top of the list of engineering materials needed for advanced design applications. This is because controlling the lamination angle can alter their structural properties and the stacking sequence leading to an optimal design. The higher specific modulus and specific strength of these composites means that the weight of certain components can be lower. The increasingly wider application to other fields of engineering has necessitated the evolution of adequate analytical tools for the better understanding of the structural behavior and efficient utilization of the materials.

In this thesis work, an analytical solution of frequency characteristics for the vibrations of rotating laminated composite cylindrical thin shells by using the “first order shear deformation theory”. Compared with classical theory and higher order theory, the “first order shear deformation theory” combines higher accuracy and lower calculation efforts. The objective of this study is to examine the effect of various shell parameters on the frequency characteristics of rotating laminated composite cross-ply thin shells. For reasons of simplicity, the simply supported boundary conditions at both ends of the shells. Figures show variation of frequency with the rotating speed. The formulation is general. Different boundary conditions, lamination schemes (which may be isotropic or orthotropic), order of shear deformation theories, and even forms of assumed solutions can be easily accommodated into the analysis.

CHAPTER - 2

LITERATURE REVIEW

CHAPTER 2

REVIEW OF LITERATURE

2.1 INTRODUCTION

The analysis of plate and shell structures has a long history starting with membrane theory and then the bending theories, of the several plate theories. Laminated composite plate analyses and shell analyses are mainly based on three theories:

(1) The classical laminated plate theory (CLPT),

(2) The first-order shear deformation theory (FSDT) and,

(3) The higher-order shear deformation theory (HSDT).

The effect of transverse shear deformation, which may be essential in some cases, is included in FSDT and HSDT, whereas it is neglected in CLPT due to the Kirchhoff hypothesis.

The classical laminate plate theory is based on the Kirchhoff hypothesis that straight lines normal to the undeformed midplane remain straight and normal to the deformed midplane and do not undergo stretching in thickness direction. These assumptions imply the vanishing of the transverse shear and transverse normal strains. The classical laminate theory has been used in the stress analysis of composite plates. However, it is only accurate for thin composite laminates.

In FSDT, a first-order displacement field is assumed for transverse strain through the thickness. Appropriate shear correction factors are required in FSDT due to the assumption of constant transverse shear strain and shear stress through the plate thickness, which is contradictory to the zero shear stress condition on the bounding planes of the laminate and actual stress states through the layer thickness.

Higher-order polynomials are used to represent displacement components through the thickness of the laminates in HSDT, and the actual transverse strain/stress through the thickness and the zero stress conditions on the top and bottom of a general laminate can be represented. A more accurate approximation of the transverse shear effect can thus be obtained with no shear correction factors. However, complexities in formulation and large computational effort make it economically unattractive. The free vibration of plates has been largely studied using first order shear deformation theories (FSDT).

2.2 REVIEW OF PLATES AND CYLINDRICAL SHELLS

During the past three to four decades, there has been continuously increasing usage of laminated composite materials in structural applications. Often encountered among these applications are plate and shell structural components. Accompanying this increasing usage has been a growth in the literature of composite laminate structural analysis, particularly for plates and to a lesser extent for circular cylindrical shells. Equations have been thoroughly developed for the deformation analysis of laminated composite plates (Ambartsumyan, 1964, 1970[1], Ashton and Whitney, 1970 [1]), as well as for circular cylindrical shells.

Now many of the existing methods of analysis for multilayered anisotropic plates and shells are direct extensions of those developed earlier for homogeneous isotropic and orthotropic plates and shells. The book written by Flugge (1934), [9] and Kraus (1967), [13] deals with both the statics and dynamics of shells. After the pioneering works of Ambartsumyan and Dong (1968) [7], there has been consistent progress in the field of anisotropic layered shells. The publications by Bert and Egle and Leissa deal with the literature on dynamic problems in laminated shells.

Thin cylindrical shells are widely used as structural elements. Studies of thin cylindrical shells are extensive and many theories have been developed. The first to study the cylindrical shell problem was Aron and the first to provide a mathematical framework for a thin shell theory was Love. Love’s mathematical framework, also known as Love’s first approximation theory, consisted of four principal assumptions under which many thin shell theories were developed. These four assumptions, commonly known as the Kirchhoff-Love hypotheses form the background of many linear thin shell theories, which over the years have been modified and employed to varying degree.

The vibration of laminated cylindrical shells has been studied to a greater extent compared to any other shell geometry. Dong (1968), [4] using Donnell's theory conducted an extensive study of the lower modes of laminated orthotropic cylindrical shells; the quantitative effects were determined by varying the material properties, pre-stress and boundary conditions. Hu and Tsuiji (1999), [11] presented a numerical method for analyzing the free vibrations of curved and twisted cylindrical thin panels by means of the principle of virtual work and the free vibration problem was solved using the Rayleigh-Ritz method, assuming two dimensional polynomial functions as displacement functions. It was shown that the method was effective in solving free vibration problems for cylindrical thin panels with curvature and twist by comparing the numerical results with previous results. The effects of curvature and twist on the frequency parameters and mode shapes were also discussed.

Lam K.Y. and Wu Qian (2000), [17] presented analytical solutions for the vibrations of thick symmetric angle-ply laminated composite cylindrical shells using the first-order shear deformation theory. A complex method was developed to deal with the partial differential governing equations of thick symmetric angle-ply laminated composite cylindrical shells. The frequency characteristics for thick symmetric angle-ply laminated composite cylindrical shells with different h/R and L/R ratios were studied in comparison with those of symmetric cross-ply laminates. Also, the influence of lamination angle and number of lamination layers on frequency was investigated in detail.

Ng T. Y., Lam K. Y (1999), [21] worked on the dynamic stability of simply-supported, isotropic cylindrical panels under combined static and periodic axial forces. An extension of Donnell's shell theory to a first-order shear deformation theory was used, and a system of Mathieu-Hill equations were obtained via a normal-mode expansion and the parametric resonance response was analyzed using Bolotin's method. Results were compared with those obtained using the classical shell theory. The effects of the thickness-to-radius ratio on the instability regions are examined in detail. Lam K. Y. and Loy C. T (1995), [18] worked on the natural frequencies of thin orthotropic laminated cylindrical shells. A straightforward method of analysis involving Love's first approximation theory and Ritz's procedure was used to study the influence of boundary conditions and fiber orientation on these shells. The boundary conditions considered in his paper were clamped-clamped, clamped-simply supported, clamped-sliding, and clamped-free. Yu S. D., Cleghorn W. L and Fenton R. G., (1995), [29] reviewed the analytical methods used and the boundary conditions encountered in the accurate free vibration analysis of open circular cylindrical shells. The simple boundary conditions associated with the Donnell Mushtari theory of thin shells were classified into primary and secondary boundary conditions. Exact solutions for basic shells with different combinations of primary boundary conditions were obtained using the generalized Navier method. Accurate solutions for shells with secondary and mixed boundary conditions are obtained by using the method of superposition.

2.3 REVIEW OF ROTATING CYLINDRICAL SHELL

Blades are often part of machinery rotating at high speed, so it is very important to ensure safety while rotating. The configuration of turbo machinery blades is complex and usually thin with a small aspect ratio, twisted in the lengthwise direction and cambered in the chord wise direction. That is the reason why so many researchers have studied them for the past few decades. In majority of cases, a turbo machinery blade is modeled as a beam. Leissa A. W. (1983), [19] presented a comparison of blade and shell models. M. A. Dokanish and Rawtani (1971), [8] used the finite element technique to determine the natural frequencies and the mode shapes of a cantilever plate mounted on the periphery of a rotating disc. The plane of the plate is assumed to make an arbitrary angle with the plane of rotation of the disc. McGhee and Chu [20] carried out a three dimensional continuum vibration analysis for rotating, laminated composite blades using Ritz method. Full geometric nonlinearity and the coriolis acceleration term were included in the blade kinematics. Bhumbia, Kosmatka [3] and Reddy studied free vibration behavior of shear deformable, composite rotating blades including geometric non linearity in the form of Von Karman strains along with plane stress assumption in the constitutive relations. Karmakar and Sinha (1995), [14] analyzed, using finite element method, the free vibration characteristics of rotating laminated composite pre-twisted cantilever plates. A nine nodded three dimensional degenerate composite shell element was developed and used for the analysis. Sivadas (1995), [24] studied circular conical shells rotating about their axis of revolution. The shells were analyzed by using moderately thick shell theory with shear deformation and rotary inertia. The natural frequencies and the damping factor due to material damping were analyzed.

Second order strains with the in plane and transverse non-linear terms were used for the derivation of the geometric matrix. An iso-parametric axis-symmetric finite element with five degrees of freedom per node was used for the solution. The effect of rotation on the frequencies of the shells was studied by incorporating the Coriolis acceleration, rotational energy, pre-stressing due to centrifugal force and torque and damping due to the material. Young-Jung Kee, Ji-Hwan Kim, (2004), [31] analyzed the vibration of a rotating composite blade. A general formulation is derived for an initially twisted rotating shell structure including the effect of centrifugal force and Coriolis acceleration. In this work, the blade was assumed to be a moderately thick open cylindrical shell that includes the transverse shear deformation and rotary inertia, and was oriented arbitrarily with respect to the axis of rotation to consider the effects of disc radius and setting angle. Based on the concept of the degenerated shell element with the Reissner–Mindlin's assumptions, the finite element method was used for solving the governing equations. In the numerical study, effects of various parameters were investigated: initial twisting angles, thickness to radius ratios, layer lamination and fiber orientation of composite blades.

In the literature, the bulk of the works on cylindrical shells are on non-rotating shells. The first recorded work on a rotating shell is that of Bryan (1890), [2] who studied the vibration of a rotating cylindrical shell by using an analysis for a spinning ring. Later works on rotating shells include the study of the Coriolis effect by Di Taranto and Lessen (1964), [27] that by Srinivasan and Lauterbach (1971), [25] for infinitely long rotating shells, and by Zohar and Aboudi (1973), [32], Wang and Chen (1974), [28] for finite length shells. Rand and Stavsky (1991), [23], Chun and Bert, (1993), [4], have also carried out Works on composite rotating cylindrical shells. Chen, Zhao and Shen (1993), [33], have presented a finite element analysis for a rotating cylindrical shell. The general equations of the vibrations of high speed rotating shells of revolution considering Coriolis accelerations and large deformations were established using the method of linear approximation. A nine node curvilinear super parametric finite element was used to solve the problems of high speed rotating shells of revolution.

Omer Civalek (May 2007), [6] dealt with the free vibration analysis of rotating laminated cylindrical shells. The analysis used discrete singular convolution (DSC) technique to determine frequencies. Regularized Shannon's delta (RSD) kernel was selected as singular convolution to illustrate the algorithm. The formulations were based on the Love's first approximation shell theory, and included the effects of initial hoop tension and centrifugal and Coriolis accelerations due to rotation. The spatial derivatives in both the governing equations and the boundary conditions were discretized by the DSC method. Frequency parameters were obtained for different types of boundary conditions, rotating velocity and geometric parameters. The effect of the circumferential node number on the vibration behavior of the shell was also analyzed. Ji-Hwan Kim (2004), [16] studied initially twisted rotating shell structures including the effect of centrifugal force and Coriolis acceleration. In his work, the blade was assumed to be a moderately thick open cylindrical shell that includes the transverse shear deformation and rotary inertia, and was oriented arbitrarily with respect to the axis of rotation to consider the effects of disc radius and setting angle.

Lee (1998), [30] gave analytical solutions for the free vibration of the rotating composite cylindrical shells with axial stiffeners (stringers) and circumferential stiffeners (rings), that is, orthogonal stiffeners, using the energy method. The cylindrical shells are stiffened at uniform intervals and the stiffeners have the same material and geometric properties. The Love's shell theory based on the discrete stiffener theory was used to derive the governing equation of the rotating composite cylindrical shell with orthogonal stiffeners. The effect of the parameters such as the stiffener's height-to-width ratio, the shell thickness-to-radius ratio and the shell length-to-radius ratio was studied. The natural frequencies were compared with the previously published analytical results for the un-stiffened rotating composite shell and the orthogonally stiffened isotropic cylindrical shells. Jafari and Bagheri (2006), [12] researched the free vibration analysis of simply supported rotating cylindrical shells with circumferential stiffeners. Ritz method was applied while stiffeners were treated as discrete elements. In strain energy formulation, by adopting Sander's theorem, stretching and bending characteristics of shells were considered. Also stretching, bending and warping effects of stiffeners were investigated. The translational inertia in three directions for shell and stiffeners, and rotary inertia for stiffeners were considered. The effects of initial hoop tension, centrifugal and Coriolis forces due to the rotation of the shell were studied. Polynomial functions were used for Ritz functions. At constant total mass of stiffeners, the effects of non-uniform eccentricity distribution and non-uniform rings spacing distribution (separately and simultaneously) on natural frequencies were investigated.

Moreover, the influence of rotating speed on natural frequencies for the so-called non-uniform stiffeners distribution was studied. In similar way, Liew (2002), [28] worked on the vibration analysis of simply supported rotating cross ply laminated cylindrical shells with axial and circumferential stiffeners, that is, stringers and rings using an energy approach. The effects of these stiffeners were evaluated via two methods, namely by a variation formulation with individual stiffeners treated as discrete elements; and by averaging method whereby the properties of the stiffeners were averaged over the shell surface.

Kim and Bolton (2004), [15] considered the effects of rotation on wave propagation within a tire's tread-band, the vibration of an inflated, circular cylindrical shell, rotating about a fixed axis. The equations of motion of the rotating shell were formulated in a fixed reference frame (i.e., Eulerian coordinates). By assuming wave-like solutions for the free vibration case, the natural frequencies and corresponding wave-like basis functions could then be obtained. A natural frequency selection procedure was introduced that can be used to associate each of the basis functions with a single natural frequency. The basis functions were then superimposed to represent the forced response of the system when driven by a point harmonic force at a fixed location in the reference frame. Kadivar and Samani (2000), [16] investigated the elasto-dynamic analysis of rotating thick composite cylindrical shells. The layer wise laminate theory was used. Unlike the equivalent single layer (ESL) theories, the layer wise theories assumed separate displacement field expansions within each material layer, providing a kinematically correct representation of the strain field in discrete layers. In deriving the governing equations, 3-D strain relations were used and the centrifugal and Coriolis forces were included in the theory. The Navier-type solutions were presented for simply supported boundary conditions. Natural frequencies of forward and backward waves were presented, showing their variation with rotating angular velocity.

Guo and Chu (2001), [10] solved the problems of the vibration of rotating cylindrical shells by using a nine-node super-parametric finite element with shear and axial deformation and rotary inertia. The non-linear plate-shell theory for large deflection was used to handle the cylindrical shell before it reached equilibrium state by centrifugal force. Hamilton's principle was used to present the motion equation in finite element form. The effects of Coriolis acceleration, centrifugal force, initial tension and geometric non-linearity due to large deformation were considered in this model. The effect of geometric non linearity due to large deformation and the effect of boundary conditions on the frequency parameter of spinning cylindrical shells and the effect of rotation speed on the different modes of spinning cylindrical shells were also investigated in detail.

Padovan (1975), [22] developed a quasi-analytical finite element procedure to obtain the frequency and buckling eigen values of pre-stressed rotating anisotropic shells of revolution. In addition to the usual centrifugal forces, the rotation effects treated also included the contribution of Coriolis forces. Furthermore, since a nonlinear version of Novozhilov's shell theory was employed to develop the element formulation, the effects of moderately large pre-stress deflection states were handled.

Zhang (2002), [34] presented the vibration analysis of rotating laminated composite cylindrical shells using the wave propagation approach. The influence of the shell parameters, the axial mode m, the circumferential mode n, the thickness-to-radius ratio h/R, the length-to-radius ratio L/R, the rotating speed X (rps) and the boundary conditions on the natural frequencies, was investigated. At low circumferential mode n, the stationary frequency was between the frequencies for forward and backward whirl modes. But at high circumferential mode n, the stationary frequency was smaller than both the forward and backward frequencies. The boundary conditions considered were clamped-clamped, clamped-simply supported, simply supported-simply supported, and clamped-sliding conditions. The influence of boundary conditions on the frequencies was more significant at small circumferential mode n. It was also found that the transition of fundamental frequency from the higher mode n curve to the lower mode n curve took place at different h/R ratios for different boundary conditions.

The natural frequencies of the forward and backward modes of thin rotating laminated cylindrical shells were determined by using four common thin shell theories, namely, Donnell's, Flugge's, Love's and Sander's theories. A unified analysis was formulated with the use of tracers so that it could be reduced to any of the four shell theories by giving appropriate values to the tracers. For simplicity, results were presented only for the case of simply supported-simply supported boundary conditions, which were satisfied by expressing the displacement fields in terms of the products of sine and cosine functions. Numerical results presented were the non-dimensional frequency parameters of the forward and backward traveling modes for rotating cylindrical shells and the non-dimensional frequency parameters for non-rotating cylindrical shells.

2.4 OBJECTIVE AND SCOPE OF PRESENT INVESTIGATION

In this paper, an analytical solution of frequency characteristics for the vibrations of rotating laminated composite cylindrical thin shells is presented by using the "First order shear deformation theory”. Compared with classical theory and higher order theory, the first order shear deformation theory combines higher accuracy and lower calculation efforts.

2.4.1 Objectives

1. To investigate the frequency characteristics for different layer configuration on the natural frequency.
2. To investigate the frequency characteristics for different geometric properties.

For reasons of simplicity, the boundary conditions are simply supported at both ends of the shells. Figures are given to show variation of frequency with the rotating speed. The formulation is general. Different boundary conditions, lamination schemes (which is cross-ply), order of shear deformation theories, and even forms of assumed solutions can be easily accommodated into the analysis.

2.4.2 Present Work

The present study is carried out to find the natural frequency of vibration of laminated orthotropic cylindrical shells that are simply supported. A first order shear deformation theory of laminated shells has been developed. The thickness coordinate multiplied by the curvature is assumed to be small in comparison to unity and hence negligible. The governing equations, including the rotary inertia are presented in chapter 3. These equations are then reduced to the equations of motion for cylindrical shell and the Navier solution has been obtained for cross-ply laminated shells. The resulting equations are suitably nondimensionalised. The Navier solution gives rise to an eigen value problem in matrix formulation. This matrix and its elements are presented in chapter 3.

In chapter 4, the eigenvalues of the coefficient matrix are obtained by standard computer program and change of sign of the determinant value is checked for values of 1% on either side of the root. The program gives the lowest value of required frequency parameter. The results are compared with earlier results for two layers and three layer cross-ply shell to check the formulation and computer program. The variation of the natural frequency for different geometric properties and layer configuration of the cylindrical shell is presented in chapter 4.

CHAPTER - 3

THEORETICAL FORMULATION

CHAPTER - 3

THEORETICAL FORMULATION

3.1 INTRODUCTION

The present study deals with the vibration of rotating thin laminated composite cylindrical shells. A first order shear deformation theory of shells is used. The displacements of the middle surface are expanded as linear functions of the thickness coordinate. In the first-order shear deformation theory (FSDT) for plates, the displacement components u, v, and w in the a, b, z directions in a laminate element can be expressed in terms of the corresponding mid-plane displacement components u^0, v^0, w^0, and the rotations $f_{1,}f_2$ of the mid-plane normal along a and b axes.

The governing equations including the effect of shear deformation are presented in orthogonal curvilinear co-ordinates for laminated orthotropic shells. These equations are then reduced to the governing equations for vibration of laminated orthotropic rotating cylindrical shells. The equations are non-dimensionalised. The Navier-type exact solution for natural vibration is presented for rotating cylindrical shells under simply supported boundary conditions. This gives rise to an eigen value problem in matrix formulation whose eigen values are the frequency parameters.

3.2 BASIC ASSUMPTIONS

A set of simplifying assumptions that provide a reasonable description of the behavior of thin elastic shells is used to derive the equilibrium equations that are consistent with the assumed displacement field:

1. No slippage takes place between the layers.
2. The effect of transverse normal stress on the gross response of the laminate is assumed to be negligible.
3. The line elements of the shell normal to the reference surface do not change their length after deformation.

4. The thickness coordinate of the shell is small compared to the principal radii of curvature (z/R_1, z/R_2 <<<1).
5. Normal to the reference surface of the shell before deformation remains straight, but not necessarily normal, after deformation (a relaxed Kirchhoff's-Love's hypothesis).

3.3 STRAIN DISPLACEMENT RELATIONS

Figure 3.1 (a) contains an element of a doubly curved shell. Here (α ,β , z) denote the orthogonal curvilinear coordinates (shell coordinates) such that α - and β -curves are lines of curvature on the mid surface, z = 0, and z-curves are straight lines perpendicular to the surface, z = 0. For the doubly curved shells discussed here, the lines of principal curvature coincide with the co-ordinate lines. The values of the principal curvature of the middle surface are denoted by R_1 and R_2.

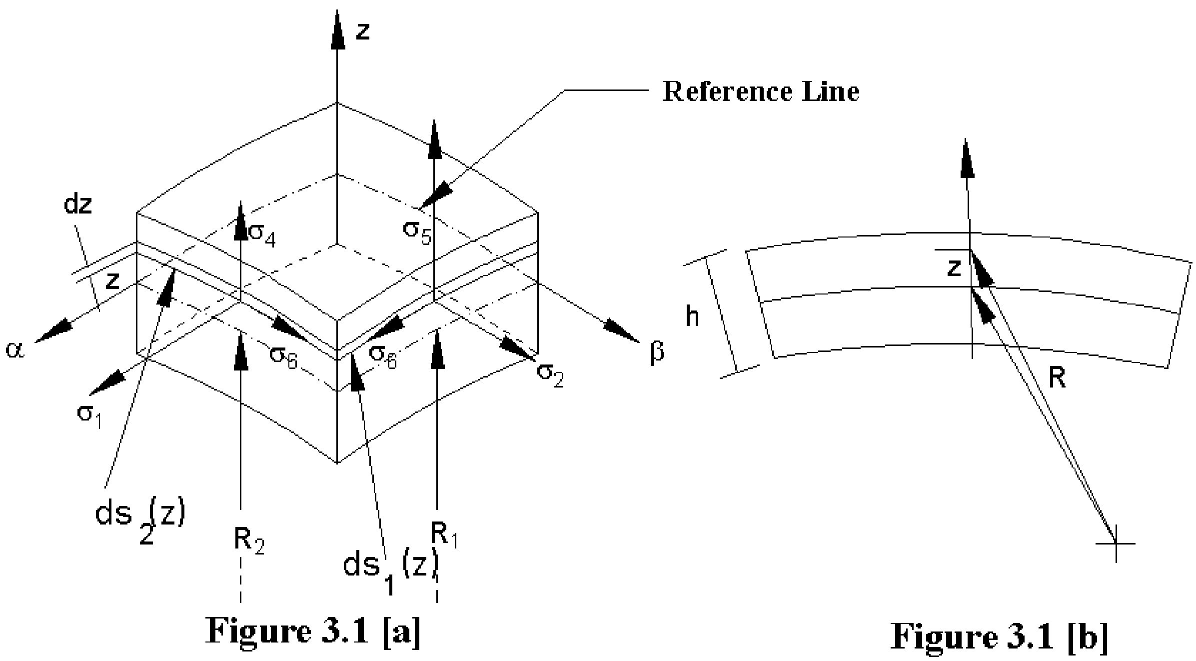


FIGURE 3.1: GEOMETRY OF LAMINATED SHELL.

The position vector of a point on the middle surface is denoted by r and the position of a point at distance, z, from the middle surface is denoted by R [see Fig. 3.1(b)]. The distance, ds, between points (α , β , z) and $(\alpha + d\alpha, \beta + d\beta, z + dz)$ is determined by

$$(ds)^2 = d\bar{r} \cdot d\bar{r} \qquad \text{.....................1}$$

$$d\bar{r} = \frac{\partial \bar{r}}{\partial \alpha} d\alpha + \frac{\partial \bar{r}}{\partial \beta} d\beta \qquad \text{.....................2}$$

The magnitude ds of $d\bar{r}$ is given in equation (2), the vectors $\frac{\partial \bar{r}}{\partial \alpha}$ and $\frac{\partial \bar{r}}{\partial \beta}$ are tangent to the α and β coordinate lines. Then equation (1) can be proceed as

$$(ds)^2 = \frac{\partial \bar{r}}{\partial \alpha} \cdot \frac{\partial \bar{r}}{\partial \alpha} (d\alpha)^2 + \frac{\partial \bar{r}}{\partial \beta} \frac{\partial \bar{r}}{\partial \beta} (\partial \beta)^2 + 2 \frac{\partial \bar{r}}{\partial \alpha} \frac{\partial \bar{r}}{\partial \beta} d\alpha d\beta \qquad \text{.....................3}$$

In the following, we limit ourselves to orthogonal curvilinear coordinates which coincide with the lines of principal curvature of the neutral surface. The third term in equation (3) thus becomes

$$2 \frac{\partial \bar{r}}{\partial \alpha} \cdot \frac{\partial \bar{r}}{\partial \beta} d\alpha \cdot d\beta = 2 \left| \frac{\partial \bar{r}}{\partial \alpha} \right| \cdot \left| \frac{\partial \bar{r}}{\partial \beta} \right| \cdot \cos \frac{\pi}{2} \cdot d\alpha \cdot d\beta = 0 \qquad \text{.....................4}$$

Where we define

$$\begin{aligned} \frac{\partial \bar{r}}{\partial \alpha} \cdot \frac{\partial \bar{r}}{\partial \alpha} &= \left| \frac{\partial \bar{r}}{\partial \alpha} \right|^2 = A_1^2 \\ \frac{\partial \bar{r}}{\partial \beta} \cdot \frac{\partial \bar{r}}{\partial \beta} &= \left| \frac{\partial \bar{r}}{\partial \beta} \right|^2 = A_2^2 \end{aligned} \qquad \text{.....................5}$$

Now the equation (3) becomes

$$(ds)^2 = A_1^2 (\partial \alpha)^2 + A_2^2 (\partial \beta)^2 \qquad \text{.....................6}$$

This equation is called the fundamental form and A_1 and A_2 are the fundamental form parameters, Lame parameters, or surface metrics. The distance, dS, between points (α, β, z) and $(\alpha + d\alpha, \beta + d\beta, z + dz)$ is given by

$$(dS)^2 = d\bar{R} \cdot d\bar{R} = L_1^2\,(d\alpha)^2 + L_2^2\,(d\beta)^2 + L_3^2\,(dz)^2 \qquad \text{.......................7}$$

In which $d\bar{R} = \left(\frac{\partial \bar{R}}{\partial \alpha}\right) \cdot d\alpha + \left(\frac{\partial \bar{R}}{\partial \beta}\right) \cdot d\beta + \left(\frac{\partial \bar{R}}{\partial z}\right) \cdot \partial z$, and L_1, L_2, and L_3 are the Lame's coefficients

$$L_1 = A_1\left(1 + \frac{z}{R_1}\right);\; L_2 = A_2\left(1 + \frac{z}{R_2}\right);\; L_3 = 1 \qquad \text{.......................8}$$

It should be noted that the vectors $\frac{\partial \bar{R}}{\partial \alpha}$ and $\frac{\partial \bar{R}}{\partial \beta}$ are parallel to the vectors $\frac{\partial \bar{r}}{\partial \alpha}$ and $\frac{\partial \bar{r}}{\partial \beta}$.

From the figure 3.1(a) the elements of area of the cross section are

$$\begin{aligned} da_1 &= L_1\, d\alpha\, dz = A_1\left(1 + \frac{z}{R_1}\right) d\alpha\, dz; \\ da_2 &= L_2\, d\beta\, dz = A_2\left(1 + \frac{z}{R_2}\right) d\beta\, dz \end{aligned} \qquad \text{.......................9}$$

The strain displacement equations of a shell are an approximation, within the assumptions made previously, of the strain displacement relations referred to orthogonal curvilinear coordinates. In addition, we assume that the transverse displacement, w, does not vary with z. As in the shear deformable theory of flat plates, we begin with the displacement field

$$\begin{aligned} \bar{u} &= \frac{1}{A_1}(L_1 u) + z\phi_1 = \frac{1}{A_1} A_1\left(1 + \frac{z}{R_1}\right) u + z\phi_1 = u + z\phi_1 \\ \bar{v} &= \frac{1}{A_2}(L_2 v) + z\phi_2 = \frac{1}{A_2} A_2\left(1 + \frac{z}{R_2}\right) v + z\phi_2 = v + z\phi_2 \\ \bar{w} &= w \end{aligned} \qquad \text{.......................10}$$

Here $(\bar{u}, \bar{v}, \bar{w})$ = the displacement of a point (α, β, z) along the (α, β z) coordinates; and (u, v, w) = the displacements of a point (α, β, 0). Now substituting equation [10] in strain displacement relations referred to an orthogonal curvilinear coordinate system, we get

$$
\begin{aligned}
\varepsilon_1 &= \varepsilon_1^0 + zk_1 \\
\varepsilon_2 &= \varepsilon_2^0 + zk_2 \\
\varepsilon_4 &= \varepsilon_4^0 \\
\varepsilon_5 &= \varepsilon_5^0 \\
\varepsilon_6 &= \varepsilon_6^0 + zk_6
\end{aligned}
\qquad \ldots\ldots\ldots\ldots\ldots\ldots 11.a
$$

Where,

$$
\begin{aligned}
&\varepsilon_1^0 = \frac{1}{A_1}\frac{\partial u}{\partial \alpha} + \frac{w}{R_1}; \varepsilon_2^0 = \frac{1}{A_2}\frac{\partial v}{\partial \beta} + \frac{w}{R_2}; \varepsilon_4^0 = \frac{1}{A_2}\frac{\partial w}{\partial \beta} + \phi_2 - \frac{v}{R_2}; \\
&\varepsilon_5^0 = \frac{1}{A_1}\frac{\partial w}{\partial \alpha} + \phi_1 - \frac{u}{R_1}; \varepsilon_6^0 = \frac{1}{A_1}\frac{\partial v}{\partial \alpha} + \frac{1}{A_2}\frac{\partial u}{\partial \beta}; k_1 = \frac{1}{A_1}\frac{\partial \phi_1}{\partial \alpha}; k_2 = \frac{1}{A_2}\frac{\partial \phi_2}{\partial \beta} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 11.b \\
&k_6 = \frac{1}{A_1}\frac{\partial \phi_2}{\partial \alpha} + \frac{1}{A_2}\frac{\partial \phi_1}{\partial \beta} + \frac{1}{2}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\left(\frac{1}{A_1}\frac{\partial v}{\partial \alpha} - \frac{1}{A_2}\frac{\partial u}{\partial \beta}\right)
\end{aligned}
$$

Where ϕ_1 and ϕ_2 are the rotation of the reference surface, z = 0, about the β -and α -coordinate axes, respectively. It should be noted that the displacement field in equation [10] can be used to derive the general theory of laminated shells.

3.4 STRESS-STRAIN RESULTANT

The stress-strain relation for the K^{th} orthotropic layer takes the following form:

$$
\begin{Bmatrix} \sigma_1^K \\ \sigma_2^K \\ \sigma_4^K \\ \sigma_5^K \\ \sigma_6^K \end{Bmatrix} = \begin{bmatrix} Q_{11}^K & Q_{12}^K & 0 & 0 & Q_{16}^K \\ Q_{12}^K & Q_{22}^K & 0 & 0 & Q_{26}^K \\ 0 & 0 & Q_{44}^K & Q_{45}^K & 0 \\ 0 & 0 & Q_{45}^K & Q_{55}^K & 0 \\ Q_{16}^K & Q_{26}^K & 0 & 0 & Q_{66}^K \end{bmatrix} \begin{Bmatrix} \varepsilon_1^k \\ \varepsilon_2^k \\ \varepsilon_4^k \\ \varepsilon_5^k \\ \varepsilon_6^k \end{Bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots 12
$$

For special orthotropic material, in which the principal axis direction coincides with the axis of the material direction,

$$Q_{16}^K = Q_{26}^K = Q_{45}^K = 0$$

Then,

$$\begin{Bmatrix} \sigma_1^K \\ \sigma_2^K \\ \sigma_4^K \\ \sigma_5^K \\ \sigma_6^K \end{Bmatrix} = \begin{bmatrix} Q_{11}^K & Q_{12}^K & 0 & 0 & 0 \\ Q_{12}^K & Q_{22}^K & 0 & 0 & 0 \\ 0 & 0 & Q_{44}^K & 0 & 0 \\ 0 & 0 & 0 & Q_{55}^K & 0 \\ 0 & 0 & 0 & 0 & Q_{66}^K \end{bmatrix} \begin{Bmatrix} \varepsilon_1^k \\ \varepsilon_2^k \\ \varepsilon_4^k \\ \varepsilon_5^k \\ \varepsilon_6^k \end{Bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 13$$

For generalized plane stress conditions, the above elastic module Q^k_{ij} is related to the usual engineering constants as follows:

$$Q_{11}^K = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{E_1\nu_{21}}{1-\nu_{12}\nu_{21}} = \frac{E_2\nu_{12}}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 14$$

$$Q_{44} = G_{13}, \qquad Q_{55} = G_{23},$$

$$Q_{66} = G_{12}, \qquad \frac{E_1}{E_2} = \frac{\nu_{12}}{\nu_{21}}$$

3.5 STRESS RESULTANTS AND STRESS COUPLES

Let N_1 be the tensile force, measured per unit length along a β-coordinate line, on a cross section perpendicular to α-coordinate line. Then the total tensile force on the differential element in the α-direction is $N_1 \cdot \beta \cdot d\beta$. This force is equal to the integral of $\sigma_1 da_2$ over the thickness

$$N_1\ \beta\ d\beta = \int_{-h/2}^{h/2} \sigma_1\ da_2\ dz \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 15$$

In which h = the thickness of the shell (z = -h/2 and z = h/2 denote the bottom and top surfaces of the shell) and da_2 is the area of cross section. Using equation (9) we can write.

$$N_1\ \beta\ d\beta = \int_{-h/2}^{h/2} \sigma_1 \left(1+\frac{z}{R_2}\right) dz \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 16$$

Similarly, the remaining stress resultants per unit length are given as

$$
\begin{Bmatrix} N_1 \\ N_2 \\ N_{12} \\ N_{21} \\ Q_1 \\ Q_2 \\ M_1 \\ M_2 \\ M_{12} \\ M_{21} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_1\left(1+\frac{z}{R_2}\right) \\ \sigma_2\left(1+\frac{z}{R_1}\right) \\ \sigma_6\left(1+\frac{z}{R_2}\right) \\ \sigma_6\left(1+\frac{z}{R_1}\right) \\ \sigma_5\left(1+\frac{z}{R_2}\right) \\ \sigma_4\left(1+\frac{z}{R_1}\right) \\ z\sigma_1\left(1+\frac{z}{R_2}\right) \\ z\sigma_2\left(1+\frac{z}{R_1}\right) \\ z\sigma_6\left(1+\frac{z}{R_2}\right) \\ z\sigma_6\left(1+\frac{z}{R_1}\right) \end{Bmatrix} dz \qquad \text{......................17}
$$

Note that, in contrast to the plate theory (which is obtained by setting $1/R_1 = 0, 1/R_2 = 0$), the shear stress resultants, N_{12} and N_{21}, and the twisting moments, M_{12} and M_{21}, are, in general, not equal. For shallow shells, however, one can neglect z/R_1 and z/R_2 in comparison with unity. Under this assumption, one has $N_{12} = N_{21} = N_6$ and $M_{12} = M_{21} = M_6$.

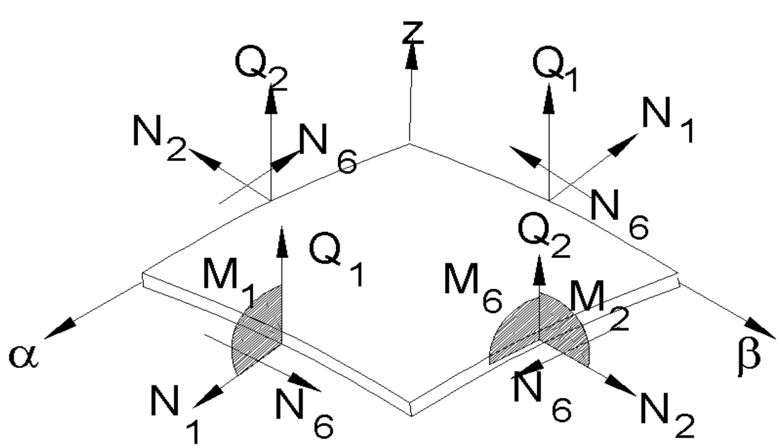


FIGURE 3.2: STRESS AND MOMENT RESULTANTS

The shell under consideration is composed of finite number of orthotropic layers of uniform thickness, as shown in Figure 3.2. In view of assumption 1, the stress resultant in equation [17] can be expressed as

$$\begin{aligned}(N_i, M_i) &= \sum_{K=1}^{N} \int_{zk-1}^{zk} \sigma_i (1, z) dz, \ldots\ldots\ldots\ldots i = 1,2,6; \\ Q_i &= \sum_{k=1}^{zk} K_i^2 \int_{zk=1}^{zk} \sigma_i dz; \ldots\ldots\ldots\ldots i = 4,5\end{aligned} \qquad \ldots\ldots\ldots\ldots 18$$

In which N = the number of layers in the shell; Z_k and Z_{k-1} = the top and bottom z-coordinates of the k^{th} lamina; and k_i = the shear correction factors.

Substituting of equation [11] and [13] into equation [18] leads to the following expression for the stress resultants and stress couples

$$\begin{aligned}N_i &= A_{ij} \varepsilon_j^0 + B_{ij} k_j \\ M_i &= B_{ij} \varepsilon_j^0 + D_{ij} k_j \\ Q_2 &= A_{44} \varepsilon_4^0 + A_{45} \varepsilon_5^0 \\ Q_1 &= A_{45} \varepsilon_4^0 + A_{55} \varepsilon_5^0\end{aligned} \qquad \ldots\ldots\ldots\ldots 19$$

Here A_{ij}, B_{ij} and D_{ij} denote the extensional, flexural-extensional coupling, and flexural stiffness. They may be defined as:

$$\begin{aligned}A_{ij} &= \sum_{k=1}^{n} (\overline{Q_{ij}})_k (z_k - z_{k-1}) \\ B_{ij} &= \frac{1}{2} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (z_k^2 - z_{k-1}^2) \\ D_{ij} &= \frac{1}{3} \sum_{k=1}^{n} (\overline{Q_{ij}})_k (z_k^3 - z_{k-1}^3);\ i, j = 1,2,6 \\ S_{ij} &= k \sum_{k=1}^{n} (\overline{Q_{ij}})_k (z_k - z_{k-1}); i, j = 4,5\end{aligned} \qquad \ldots\ldots\ldots\ldots 20$$

For i, j = 1, 2, 4, 5, 6. And h_k and h_{k+1} are the distances measured as shown in figure 3.1

3.6 GOVERNING EQUATIONS

The governing differential equations, the strain energy due to loads, kinetic energy and formulations of the general dynamic problem are derived on the basis of Hamilton's principle.

3.6.1 Governing Differential Equations

The equation of motion is obtained by taking a differential element of shell as shown in Figure 3.2 The figure shows an element with internal forces like membrane (N_1, N_2, and N_6), shearing forces (Q_1, and Q_2) and the moment resultants (M_1, M_2 and M_6).

3.6.1.1 Strain energy

The strain energy of a differential shell element can be written as

$$U = \frac{1}{2}\int_{\alpha}\int_{\beta}\int_{z}[\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_6\varepsilon_6 + \sigma_4\varepsilon_4 + \sigma_5\varepsilon_5]dV \qquad \text{.....................21}$$

dV = Volume of shell element

$$dV = A_1\left(1+\frac{z}{R_1}\right)A_2\left(1+\frac{z}{R_2}\right)d\alpha d\beta dz$$

$$dV = A_1 A_2 d\alpha d\beta dz \qquad . \qquad \text{.....................22}$$

As Z / R < < < 1

It may be easily verified that the variation that the variation of strain energy U is given by

$$\delta U = \int_{\alpha}\int_{\beta}\int_{z}[\sigma_1\delta\varepsilon_1 + \sigma_2\delta\varepsilon_2 + \sigma_6\delta\varepsilon_6 + \sigma_4\delta\varepsilon_4 + \sigma_5\delta\varepsilon_5]dV \qquad \text{.....................23}$$

Now equation [23] is independent of the material property. Substituting the variation of strain function we get,

$$U=\frac{1}{2}\int_{\alpha}\int_{\beta}\int_{z}[\sigma_1\left(\varepsilon_1^0+zk_1\right)+\sigma_2\left(\varepsilon_2^0+zk_2\right)+\sigma_6\left(\varepsilon_6^0+zk_6\right)+\sigma_4\varepsilon_4^0+\sigma_5\varepsilon_5^0]A_1A_2d\alpha d\beta dz$$

$$U=\frac{1}{2}\int_{\alpha}\int_{\beta}\int_{z}[\sigma_1\varepsilon_1^0+\sigma_1 zk_1+\sigma_2\varepsilon_2^0+\sigma_2 zk_2+\sigma_6\varepsilon_6^0+\sigma_6 zk_6+\sigma_4\varepsilon_4^0+\sigma_5\varepsilon_5^0]A_1A_2d\alpha d\beta dz \quad .....24$$

$$U=\frac{1}{2}\int_{\alpha}\int_{\beta}[N_1\varepsilon_1^0+M_1k_1+N_2\varepsilon_2^0+M_2k_2+N_6\varepsilon_6^0+M_6k_6+Q_2\varepsilon_4^0+Q_1\varepsilon_5^0]A_1A_2d\alpha d\beta$$

The variation of strain energy is given as:

$$\delta U=\int_{\alpha}\int_{\beta}[N_1\delta\varepsilon_1^0+M_1\delta k_1+N_2\delta\varepsilon_2^0+M_2\delta k_2+N_6\delta\varepsilon_6^0+M_6\delta k_6+Q_2\delta\varepsilon_4^0+Q_1\delta\varepsilon_5^0]A_1A_2d\alpha d\beta$$

Substituting for $\varepsilon_1^0,\varepsilon_2^0,\varepsilon_4^0,\varepsilon_5^0,\varepsilon_6^0,k_1,k_2,k_6$ from equation [11]

$$\begin{aligned}\delta U=\int_{\alpha}\int_{\beta}\{\quad & N_1\left(\frac{1}{A_1}\frac{\partial\delta u}{\partial\alpha}+\frac{\delta w}{R_1}\right)+M_1\frac{1}{A_1}\frac{\partial\delta\phi_1}{\partial\alpha}+N_2\left(\frac{1}{A_2}\frac{\partial\delta v}{\partial\beta}+\frac{\delta w}{R_2}\right)+M_2\frac{1}{A_2}\frac{\partial\delta\phi_2}{\partial\beta}\\ & N_6\left(\frac{1}{A_1}\frac{\partial\delta v}{\partial\alpha}+\frac{1}{A_2}\frac{\partial\delta u}{\partial\beta}\right)+M_6\left[\frac{1}{A_1}\frac{\partial\delta\phi_2}{\partial\alpha}+\frac{1}{A_2}\frac{\partial\delta\phi_1}{\partial\beta}+\frac{1}{2}\left(\frac{1}{R_2}-\frac{1}{R_1}\right)\left(\frac{1}{A_1}\frac{\partial\delta v}{\partial\alpha}-\frac{1}{A_2}\frac{\partial\delta u}{\partial\beta}\right)\right] \quad ......25\\ & +Q_2\left(\frac{1}{A_2}\frac{\partial\delta w}{\partial\beta}+\delta\phi_2-\frac{\delta v}{R_2}\right)+Q_1\left(\frac{1}{A_1}\frac{\partial\delta w}{\partial\alpha}+\delta\phi_1-\frac{\delta u}{R_1}\right)\quad\}A_1A_2d\alpha\cdot d\beta\end{aligned}$$

The above equation contains the derivative of displacement that is $\frac{\partial\delta u}{\partial\alpha},\frac{\partial\delta v}{\partial\beta}$ etc.

Integrating by parts,

$$\int_{\alpha}\int_{\beta}N_1\left(\frac{1}{A_1}\frac{\partial\delta u}{\partial\alpha}\right)A_1A_2d\alpha d\beta=\oint_{\beta}N_1\cdot A_2\cdot\delta u\cdot d\beta-\int_{\alpha}\int_{\beta}\frac{\partial\left(N_1A_2\right)}{\partial\alpha}\delta u\cdot d\alpha\cdot d\beta$$

$$\int_{\alpha}\int_{\beta}M_1\left(\frac{1}{A_1}\frac{\partial\delta\phi_1}{\partial\alpha}\right)A_1A_2d\alpha d\beta=\oint_{\beta}M_1A_2\delta\phi_1 d\beta-\int_{\alpha}\int_{\beta}\frac{\partial\left(M_1A_2\right)}{\partial\alpha}\delta\phi_1\cdot d\alpha\cdot d\beta$$

$$\int_{\alpha}\int_{\beta}N_2\left(\frac{1}{A_2}\frac{\partial\delta v}{\partial\beta}\right)A_1A_2d\alpha d\beta=\oint_{\alpha}N_2\cdot A_1\cdot\delta v\cdot d\alpha-\int_{\alpha}\int_{\beta}\frac{\partial\left(N_2A_1\right)}{\partial\beta}\delta v\cdot d\alpha\cdot d\beta$$

$$\int_{\alpha}\int_{\beta}M_2\left(\frac{1}{A_2}\frac{\partial\delta\phi_2}{\partial\beta}\right)A_1A_2d\alpha d\beta=\oint_{\alpha}M_2\cdot A_1\cdot\delta\phi_2\cdot d\alpha-\int_{\alpha}\int_{\beta}\frac{\partial\left(M_2A_1\right)}{\partial\beta}\delta\phi_2\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta N_6\left(\frac{1}{A_1}\frac{\partial\delta v}{\partial\alpha}\right)A_1A_2\,d\alpha d\beta = \oint_\beta N_6\cdot A_2\cdot\delta v\cdot d\beta - \int_\alpha\int_\beta\frac{\partial(N_6A_2)}{\partial\alpha}\delta v\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta N_6\left(\frac{1}{A_2}\frac{\partial\delta u}{\partial\beta}\right)A_1A_2\,d\alpha d\beta = \oint_\alpha N_6\cdot A_1\cdot\delta u\cdot d\alpha - \int_\alpha\int_\beta\frac{\partial(N_6A_1)}{\partial\beta}\delta u\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta M_6\left(\frac{1}{A_1}\frac{\partial\delta\phi_2}{\partial\alpha}\right)A_1A_2\,d\alpha d\beta = \oint_\beta M_6\cdot A_2\cdot\delta\phi_2\cdot d\beta - \int_\alpha\int_\beta\frac{\partial(M_6A_2)}{\partial\alpha}\delta\phi_2\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta M_6C_0\left(\frac{1}{A_1}\frac{\partial\delta v}{\partial\alpha}\right)A_1A_2\,d\alpha d\beta = \oint_\beta C_0M_6\cdot A_2\cdot\delta v\cdot d\beta - \int_\alpha\int_\beta C_0\frac{\partial(M_6A_2)}{\partial\alpha}\delta v\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta M_6C_0\left(\frac{1}{A_2}\frac{\partial\delta u}{\partial\beta}\right)A_1A_2\,d\alpha d\beta = \oint_\alpha C_0M_6\cdot A_1\cdot\delta u\cdot d\alpha - \int_\alpha\int_\beta C_0\frac{\partial(M_6A_1)}{\partial\beta}\delta u\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta Q_2\left(\frac{1}{A_2}\frac{\partial\delta w}{\partial\beta}\right)A_1A_2\,d\alpha d\beta = \oint_\alpha Q_2\cdot A_1\cdot\delta w\cdot d\alpha - \int_\alpha\int_\beta\frac{\partial(Q_2A_1)}{\partial\beta}\delta w\cdot d\alpha\cdot d\beta$$

$$\int_\alpha\int_\beta Q_1\left(\frac{1}{A_1}\frac{\partial\delta w}{\partial\alpha}\right)A_1A_2\,d\alpha d\beta = \oint_\beta Q_1\cdot A_2\cdot\delta w\cdot d\beta - \int_\alpha\int_\beta\frac{\partial(Q_1A_2)}{\partial\alpha}\delta w\cdot d\alpha\cdot d\beta \qquad \ldots\ldots\ldots\ldots..26$$

Substituting in equation [25] we get

$$\begin{aligned}\delta U = \int_\alpha\int_\beta\{ & -\frac{\partial}{\partial\alpha}(N_1A_2)\delta u - \frac{\partial}{\partial\beta}(N_6A_1)\delta u + \frac{\partial}{\partial\beta}(M_6A_1)\delta u - \frac{Q_1A_1A_2}{R_1}\delta u \\ & - \frac{\partial}{\partial\beta}(N_2A_2)\delta v - \frac{\partial}{\partial\alpha}(N_6A_2)\delta v + C_0\frac{\partial}{\partial\alpha}(M_6A_2)\partial v - \frac{Q_2A_1A_2}{R_2}\partial v \\ & + \frac{N_1A_1A_2}{R_1}\delta w + \frac{N_2A_1A_2}{R_2}\delta w - \frac{\partial}{\partial\beta}(Q_2A_1)\delta w - \frac{\partial}{\partial\alpha}(Q_1A_2)\delta w \\ & - \frac{\partial}{\partial\alpha}(M_1A_2)\delta\phi_1 - \frac{\partial}{\partial\beta}(M_6A_1)\delta\phi_1 + Q_1A_1A_2\delta\phi_1 \qquad \ldots\ldots\ldots 27 \\ & - \frac{\partial}{\partial\beta}(M_2A_1)\delta\phi_2 - \frac{\partial}{\partial\alpha}(M_6A_2)\delta\phi_2 + Q_2A_1A_2\delta\phi_2\}\,d\alpha\cdot d\beta \\ & + \oint_\alpha(N_6\delta u + C_0M_6\delta u + N_2\delta v + Q_2\delta w + M_6\delta\phi_1 + M_2\delta\phi_2)A_2\cdot d\beta \\ & + \oint_\beta(N_1\delta u + C_0M_6\delta v + N_6\delta v + Q_1\delta w + M_1\delta\phi_1 + M_6\delta\phi_2)A_1\cdot d\alpha\end{aligned}$$

3.6.1.2 Kinetic energy

If $\overline{U}$ be the displacement vector, the kinetic energy of the shell element is given by

$$T = \frac{1}{2}\int_{v} \rho\, \dot{\overline{U}} \cdot \dot{\overline{U}}\, dV \qquad \text{………………..28}$$

Where, ρ is the mass density and

$$\dot{\overline{U}} = \left(\dot{u}\bar{i} + \dot{v}\bar{j} + \dot{w}\bar{k}\right) + \left(-\Omega\bar{w}\bar{j} + \Omega\bar{v}\bar{k}\right) = \dot{u}\bar{i} + (\dot{v} - \Omega\bar{w})\, j + \left(\dot{w} + \Omega\bar{v}\right)\bar{k} \qquad \text{.....……………...29}$$

Equation [29] represents the dot product of $\overline{U}$. Now substituting for $\bar{u}, \bar{v}, \bar{w}$ from equation [14], the above equation reduces to

$$T = \frac{\rho}{2}\int_{v} \dot{\overline{U}} \cdot \dot{\overline{U}} \cdot dV$$

$$T = \frac{\rho}{2}\int_{\alpha}\int_{\beta}\int_{z}\left(\dot{u}^2 + \left(\dot{v} - \Omega w\right)^2 + \left(\dot{w} + \Omega v\right)^2\right)\cdot d\alpha \cdot d\beta \cdot dz$$

$$T = \frac{\rho}{2}\int_{\alpha}\int_{\beta}\int_{z}\left(\left(\dot{u} + z\dot{\phi}_1\right)^2 + \left(\dot{v} + z\dot{\phi}_2 - \Omega w\right)^2 + \left(\dot{w} + \Omega v\right)^2\right)\cdot d\alpha \cdot d\beta \cdot dz$$

$$T = \frac{\rho}{2}\int_{\alpha}\int_{\beta}\int_{z}\left\{\begin{array}{l}\left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right) + z^2\left(\dot{\phi}_1^2 + \dot{\phi}_2^2\right) + \left(2z\dot{u}\dot{\phi}_1 + 2z\dot{v}\dot{\phi}_2\right) \\ + \left(-2\dot{v}\Omega w - 2z\dot{\phi}_2\Omega w + 2\dot{w}\Omega v\right) + \Omega^2\left(w^2 + v^2\right)\end{array}\right\} d\alpha \cdot d\beta \cdot dz$$

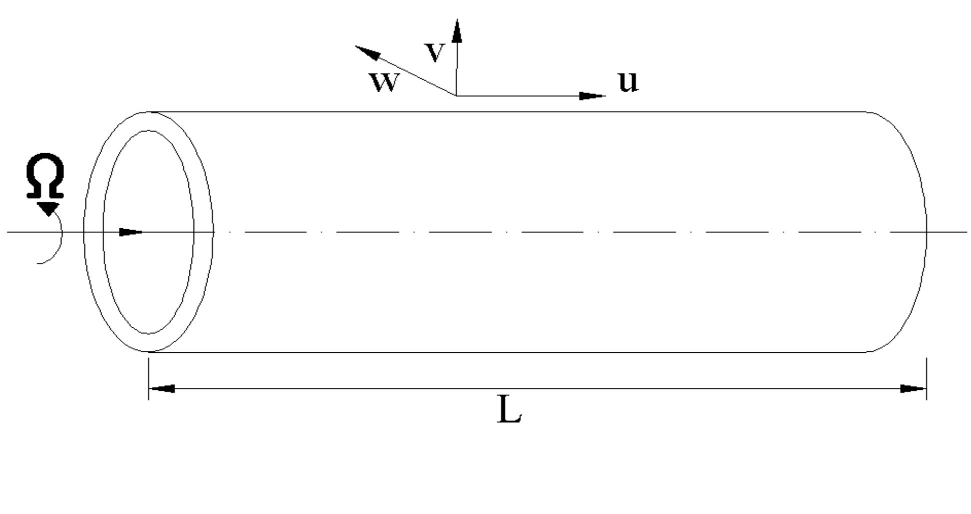


FIGURE 3.3: GEOMETRY AND CO-ORDINATE SYSTEM

Integrating over the thickness of the shell (z = -h/2 to z = h/2). And neglecting the Coriolis Effect and centrifugal effect, the kinetic energy is as given below

$$T = \frac{\rho h}{2} \iint_{\alpha\,\beta} \left\{ \dot{u}^2 + \dot{v}^2 + \dot{w}^2 + \frac{h^2}{12}\left[\dot{\phi}_1^{\,2} + \dot{\phi}_2^{\,2}\right] + \Omega^2\left(w^2 + v^2\right) \right\} A_1 \cdot A_2 \cdot d\alpha \cdot d\beta \quad \ldots\ldots\ldots\ldots 30$$

The variation of kinetic energy is given as

$$\delta T = \frac{\rho h}{2} \iint_{\alpha\,\beta} \left[\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w} + \frac{h^2}{12}\left(\dot{\phi}_1\delta\dot{\phi}_1 + \dot{\phi}_2\delta\dot{\phi}_2\right) + \Omega^2\left(w\delta w + v\delta v\right) \right] A_1 A_2\, d\alpha\, d\beta \quad \ldots\ldots\ldots\ldots 31$$

Equation [31] contains time derivatives of the variation i.e. •• etc. To eliminate these terms equation [31] is integrated by parts to obtain

$$\begin{aligned} \int_{t_1}^{t_2} \delta T = {} & \rho h \iint \left[\dot{u}\delta u + \dot{v}\delta v + \dot{w}\delta w + \frac{h^2}{12}\left(\dot{\phi}_1\delta\dot{\phi}_1 + \dot{\phi}_2\delta\dot{\phi}_2\right) \right]_{t_1}^{t_2} A_1 A_2 \cdot d\alpha \cdot d\beta \; - \\ & - \rho h \int_{t_1}^{t_2}\iint_{\alpha\,\beta} \left\{ \left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right) + \frac{h^2}{12}\left(\ddot{\phi}_1\delta\phi_1 + \ddot{\phi}_2\delta\phi_2\right) + \Omega^2\left(w\delta w + v\delta v\right) \right\} A_1 A_2 \cdot d\alpha \cdot d\beta \end{aligned} \quad \ldots\ldots\ldots\ldots 32$$

The variations of limits t = t_1 and t = t_2 must vanish. Thus the above equation reduces to

$$\int_{t_1}^{t_2} \delta T = -\rho h \int_{t_1}^{t_2}\iint_{\alpha\,\beta} \left\{ \left(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w\right) + \frac{h^2}{12}\left(\ddot{\phi}_1\delta\phi_1 + \ddot{\phi}_2\delta\phi_2\right) + \Omega^2\left(w\delta w + v\delta v\right) \right\} A_1 A_2 \cdot d\alpha \cdot d\beta \quad \ldots\ldots\ldots 33$$

3.6.2 HAMILTON'S PRINCIPLE

The equations of equilibrium are derived by applying the dynamic version of the principle of virtual work that is the Hamilton's Principle.

It states that among the set of all admissible configurations of system, the actual motion makes the quantity $\int_{t_1}^{t_2} L \; dt$ stationary, provided the configuration is known at the limits t = t_1 and t = t_2. Mathematically this means $\delta \int_{t_1}^{t_2} L \; dt = 0$

Here, L is called Lagrangian and is equal to

$$L = T - (U - V) \qquad \ldots\ldots\ldots\ldots\ldots\ldots 34$$

Where, T = Kinetic energy

U= Strain energy

V= potential of all applied loads

δ = Mathematical operation called variation. It is analogous to partial differentiation.

It is clear from equation [34] that the Lagrangian consists of kinetic, strain energy and potential of applied loads. Substituting for strain energy and kinetic energy, equation [34] becomes

$$L = \frac{\rho h}{2}\int_{\alpha}\int_{\beta}\left\{\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2 + \frac{h^2}{12}\left[\left(\frac{\partial \phi_1}{\partial t}\right)^2 + \left(\frac{\partial \phi_2}{\partial t}\right)^2\right] + \Omega^2\left(w^2 + v^2\right)\right\}A_1 A_2 \, d\alpha \, d\beta$$
$$- \frac{1}{2}\int_{\alpha}\int_{\beta}[N_1\varepsilon_1^0 + M_1 k_1 + N_2\varepsilon_2^0 + M_2 k_2 + N_6\varepsilon_6^0 + M_6 k_6 + Q_2\varepsilon_4^0 + Q_1\varepsilon_5^0]A_1 A_2 \, d\alpha \, d\beta \qquad \ldots\ldots 35$$

Now according to Hamilton's principle

$$\delta \int_{t_1}^{t_2} L \; dt = \delta \int_{t_1}^{t_2} (T - U)\, dt = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots 36$$

3.6.3 EQUATION OF EQUILIBRIUM

By applying the dynamic version of the principle of virtual work (Hamilton's Principle) and substituting the parameters of strain energy and the kinetic energy as given in equation [36] and integrating the displacement gradients by parts, the resulting equation is

$$
\begin{aligned}
&\int_{t_1}^{t_2}\int_{\alpha}\int_{\beta}\Big\{\left(\frac{\partial}{\partial\alpha}(N_1A_2)+\frac{\partial}{\partial\beta}(N_6A_1)+C_0\frac{\partial}{\partial\beta}(M_6A_1)+\frac{Q_1A_1A_2}{R_1}-\rho hA_1A_2\ddot{u}\right)\delta u\\
&+\left(\frac{\partial}{\partial\beta}(N_2A_1)+\frac{\partial}{\partial\alpha}(N_6A_2)-C_0\frac{\partial}{\partial\alpha}(M_6A_2)+\frac{Q_2A_1A_2}{R_2}-\rho hA_1A_2\left(\ddot{v}-\Omega^2v\right)\right)\delta v\\
&+\left(-\frac{N_1A_1A_2}{R_1}-\frac{N_2A_1A_2}{R_2}+\frac{\partial}{\partial\beta}(Q_2A_1)+\frac{\partial}{\partial\alpha}(Q_1A_2)-\rho hA_1A_2\left(\ddot{w}-\Omega^2w\right)\right)\delta w\\
&+\left(\frac{\partial}{\partial\alpha}(M_1A_2)+\frac{\partial}{\partial\beta}(M_6A_1)-Q_1A_1A_2-\frac{\rho h^3}{12}\ddot{\phi}_1A_1A_2\right)\delta\phi_1\\
&+\left(\frac{\partial}{\partial\beta}(M_2A_1)+\frac{\partial}{\partial\alpha}(M_6A_2)-Q_2A_1A_2-\frac{\rho h^3}{12}\ddot{\phi}_2A_1A_2\right)\delta\phi_2\Big\}\cdot d\alpha\cdot d\beta\cdot dt\\
&+\oint_{\alpha}\left(N_6\delta u+C_0M_6\delta u+N_2\delta v+Q_2\delta w+M_6\delta\phi_1+M_2\delta\phi_2\right)A_2\cdot d\beta\cdot dt\\
&+\oint_{\beta}\left(N_1\delta u+C_0M_6\delta v+N_6\delta v+Q_1\delta w+M_1\delta\phi_1+M_6\delta\phi_2\right)A_1\cdot d\alpha\cdot dt\quad=0
\end{aligned}
\qquad \text{………...…...37}
$$

Integrating by parts in the resulting equation and setting the coefficient of δu, δv, δw, $\delta\phi_1$, $\delta\phi_2$ to zero separately, the following equations of equilibrium are obtained

$$
\begin{aligned}
&\frac{\partial}{\partial\alpha}(N_1A_2)+\frac{\partial}{\partial\beta}(N_6A_1)+C_0\frac{\partial}{\partial\beta}(M_6A_1)+\frac{Q_1A_1A_2}{R_1}-\rho hA_1A_2\ddot{u}=0\\
&+\frac{\partial}{\partial\beta}(N_2A_1)+\frac{\partial}{\partial\alpha}(N_6A_2)-C_0\frac{\partial}{\partial\alpha}(M_6A_2)+\frac{Q_2A_1A_2}{R_2}-\rho hA_1A_2\left(\ddot{v}-\Omega^2v\right)=0\\
&-\frac{N_1A_1A_2}{R_1}-\frac{N_2A_1A_2}{R_2}+\frac{\partial}{\partial\beta}(Q_2A_1)+\frac{\partial}{\partial\alpha}(Q_1A_2)-\rho hA_1A_2\left(\ddot{w}-\Omega^2w\right)=0\\
&+\frac{\partial}{\partial\alpha}(M_1A_2)+\frac{\partial}{\partial\beta}(M_6A_1)-Q_1A_1A_2-\frac{\rho h^3}{12}\ddot{\phi}_1A_1A_2=0\\
&+\frac{\partial}{\partial\beta}(M_2A_1)+\frac{\partial}{\partial\alpha}(M_6A_2)-Q_2A_1A_2-\frac{\rho h^3}{12}\ddot{\phi}_2A_1A_2=0
\end{aligned}
\qquad \text{..……….………38}
$$

3.6.3.1 Cylindrical Shells

The governing equations derived in orthogonal curvilinear coordinates in the previous section for general shell element is reduced for circular cylindrical shell. The equation of motion is represented in terms of displacements. The natural frequencies of specially orthotropic laminated cylindrical shells having simply supported edges are calculated.

3.6.3.2 Equations of Equilibrium for Rotating Laminated Cylindrical Shell

For the cylindrical shell configuration shown in figure (3.3), the co-ordinates are given by $\alpha = \frac{x}{R}$, $\beta = \beta$, the Lame parameters $A_1 = A_2 = R$ and the principal curvatures R_1 = Infinity $R_2 = R$, where 'R' is the radius of the mid-surface of the cylindrical shell. Neglecting the C_0 term as it is very small as compare to unity.

Then the equation of motion in terms of the stress resultants and stress couples are obtained from Equations [38]. The equations of motion are given as

$$\frac{\partial N_1}{\partial \alpha} + \frac{\partial N_6}{\partial \beta} = \rho h R \ddot{u} \qquad \text{.........................I}$$

$$\frac{\partial N_2}{\partial \beta} + \frac{\partial N_6}{\partial \alpha} + Q_2 = \rho h R \ddot{v} - \rho h R \Omega^2 v \ . \qquad \text{.........................II}$$

$$\frac{\partial Q_1}{\partial \alpha} + \frac{\partial Q_2}{\partial \beta} - N_2 = \rho h R \ddot{w} - \rho h R \Omega^2 w \qquad \text{. .... III}$$

$$\frac{\partial M_1}{\partial \alpha} + \frac{\partial M_6}{\partial \beta} - Q_1 R = \frac{\rho h^3}{12} \ddot{\psi}_1 R.. \qquad \text{.......... .... IV} \qquad \text{.....................39}$$

$$\frac{\partial M_2}{\partial \beta} + \frac{\partial M_6}{\partial \alpha} - Q_2 R = \frac{\rho h^3}{12} \ddot{\psi}_2 R. \qquad . \qquad \text{.......... .... V}$$

The strain displacement relations [11] are substituted in the equations for the stress resultants and stress couples given in equation [19]. Since the solution for the equations of motion is done by using the Navier solution, therefore such a solution exist only for specially orthotropic shell for which the following laminate stiffness are zero.

$$D_{16} = D_{26} = B_{16} = B_{26} = A_{26} = A_{26} = 0$$

The expression for the stress resultants and stress couples so obtained are then substituted into the equation of motion [39] for specially orthotropic cylindrical shells. The equation of motion in terms of the displacements hence reduces to

$$A_{11}\frac{\partial^2 u}{\partial\alpha^2}+A_{66}\frac{\partial^2 u}{\partial\beta^2}+\left[A_{12}+A_{66}+\frac{1}{R}B_{66}\right]\frac{\partial^2 v}{\partial\alpha\cdot\partial\beta}+A_{12}\frac{\partial w}{\partial\alpha}$$
$$+B_{11}\frac{\partial^2\phi_1}{\partial\alpha^2}+B_{66}\frac{\partial^2\phi_1}{\partial\beta^2}+\left[B_{12}+B_{66}\right]\frac{\partial^2\phi_2}{\partial\alpha\cdot\partial\beta}=\rho h\cdot R^2\cdot\ddot{u}$$

$$\left[A_{66}+A_{21}+\frac{B_{21}}{R}+\frac{B_{66}}{R}\right]\frac{\partial^2 u}{\partial\alpha\partial\beta}+\left[A_{66}+2\frac{B_{66}}{R}+\frac{D_{66}}{R^2}\right]\frac{\partial^2 v}{\partial\alpha^2}+\left[A_{22}+2\frac{B_{22}}{R}+\frac{D_{22}}{R^2}\right]\frac{\partial^2 v}{\partial\beta^2}$$
$$+\left[\frac{B_{22}}{R}+A_{22}\right]\frac{\partial w}{\partial\beta}+\left[B_{66}+B_{21}+\frac{D_{66}}{2R}+\frac{D_{21}}{R}\right]\frac{\partial^2\phi_1}{\partial\alpha\cdot\partial\beta}+\left[B_{66}+\frac{D_{66}}{R}\right]\frac{\partial\phi_2}{\partial\alpha^2}$$
$$+\left[B_{22}+\frac{D_{22}}{R}\right]\frac{\partial\phi_2}{\partial\beta^2}=\rho h\cdot R^2\cdot\left(\ddot{v}-\Omega^2 v\right)$$

$$A_{21}\frac{\partial u}{\partial\alpha}-\left[A_{44}+A_{22}+\frac{B_{22}}{R}\right]\frac{\partial v}{\partial\beta}+A_{55}\frac{\partial^2 w}{\partial\alpha^2}+A_{44}\frac{\partial^2 w}{\partial\beta^2}+A_{22}w+\left[A_{55}-B_{11}\right]\frac{\partial\phi_1}{\partial\alpha}$$
$$+\left[A_{44}R-B_{12}\right]\frac{\partial\phi_2}{\partial\beta}=\rho h\cdot R^2\cdot\left(\ddot{w}-\Omega^2 w\right)$$

$$B_{11}\frac{\partial^2 u}{\partial\alpha^2}+B_{66}\frac{\partial^2 u}{\partial\beta^2}+\left(B_{12}+B_{66}+\frac{D_{12}}{R}+\frac{D_{66}}{R}\right)\frac{\partial^2 v}{\partial\alpha\partial\beta}+\left[B_{11}-A_{55}R\right]\frac{\partial w}{\partial\alpha}+D_{11}\frac{\partial^2{}_1}{\partial\alpha^2}$$
$$+D_{66}\frac{\partial^2\phi_1}{\partial\beta^2}-A_{55}R\phi_1+\left(D_{12}+D_{66}\right)\frac{\partial^2\phi_2}{\partial\alpha\cdot\partial\beta}=\frac{\rho h^3}{12}\cdot\ddot{\phi}_1\cdot R^2$$

$$\left(B_{66}+B_{21}\right)\frac{\partial^2 u}{\partial\alpha\partial\beta}+\left(B_{66}+\frac{D_{66}}{R}\right)\frac{\partial^2 v}{\partial\alpha^2}+\left[B_{22}+\frac{D_{22}}{R}\right]\frac{\partial^2 v}{\partial\beta^2}+\left[B_{22}-A_{44}R\right]\frac{\partial w}{\partial\beta}$$
$$+\left[D_{66}+D_{21}\right]\frac{\partial^2\phi_1}{\partial\alpha\partial\beta}+D_{66}\frac{\partial^2\phi_2}{\partial\alpha^2}+D_{22}\frac{\partial^2\phi_2}{\partial\beta^2}-A_{44}R^2\phi_2=\frac{\rho h^3}{12}\cdot\ddot{\phi}_2\cdot R^2 \qquad \text{..........................40}$$

3.6.4 Boundary Condition

Up to now, the analysis has been general without reference to the boundary conditions. For reasons of simplicity, only simply supported boundary condition are considered along all edges for the rotating shell. The boundary conditions for simply supported cylindrical shell are obtained as given below.

$N_1 = 0$, $v = 0$, $w = 0$,

Following the Navier solution procedure, the following solution form which satisfies the boundary conditions in equations is assumed:

$$
\begin{aligned}
u &= U \cdot \cos \lambda_m \alpha \cdot \sin n\beta \cdot e^{i\omega t} \\
v &= V \cdot \sin \lambda_m \alpha \cdot \cos n\beta \cdot e^{i\omega t} \\
w &= W \cdot \sin \lambda_m \alpha \cdot \sin n\beta \cdot e^{i\omega t} \\
\phi_1 &= \Phi_1 \cdot \cos \lambda_m \alpha \cdot \sin n\beta \cdot e^{i\omega t} \\
\phi_2 &= \Phi_2 \cdot \sin \lambda_m \alpha \cdot \cos n\beta \cdot e^{i\omega t}
\end{aligned}
\qquad \text{.....……………..41}
$$

where $\lambda_m = \dfrac{m\pi R}{L}$, and U, V, W, Φ_1 and Φ_2 are the maximum amplitudes, m and n are known as the axial half wave number and circumferential wave number respectively. This implies during vibration, the shell generators are assumed to subdivide into m half waves and the circumferences subdivide into 2n half waves (Figure 3.4)

Introducing the expressions [41] into the governing equation of motion in terms of displacements [40]. The following equation in matrix form is obtained, which is general eigen value problem.

$$[C]\cdot\{X\} = \bar{\omega}^2 \cdot [M]\{X\} \qquad \text{......…………….43}$$

Where,

ω^2 is the eigenvalue

{X} is a column matrix of amplitude of vibration or eigenvector.

[C] is 5 x 5 matrices. The coefficient of the matrices describe in Appendix.

[M] is the 5 x 5 matrices having only diagonal element in the matrice.

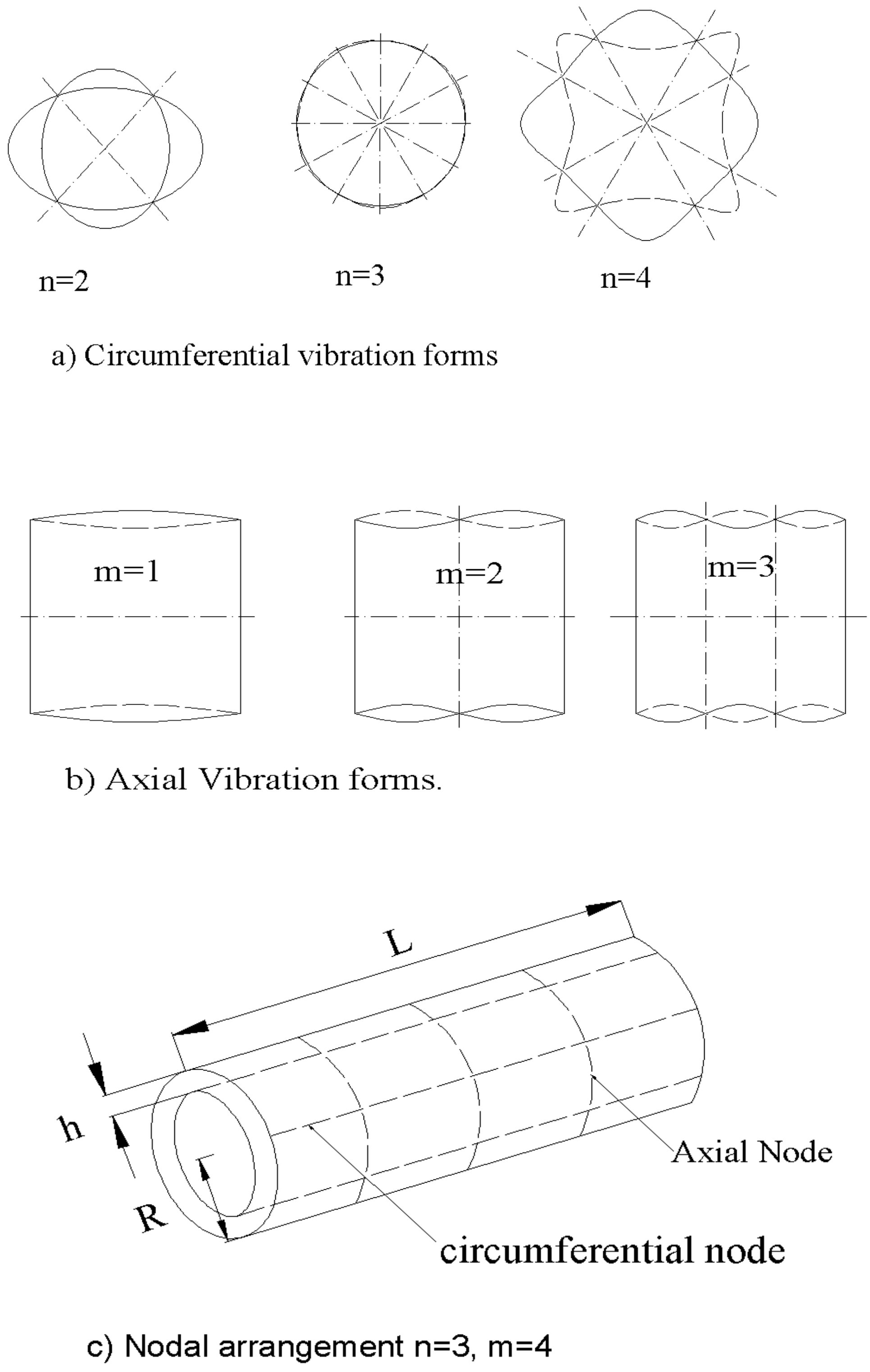


FIGURE 3.4: VIBRATION FORMS FOR CIRCULAR CYLINDRICAL SHELLS

For convenience, the elements of the above matrices are suitably non-dimensionalised as follows

$$
\begin{aligned}
&u = \bar{u} \cdot h \quad , \quad U = \bar{U} \cdot h \\
&v = \bar{v} \cdot h \quad , \quad V = \bar{V} \cdot h \\
&w = \bar{w} \cdot h \quad , \quad W = \bar{W} \cdot h \\
&\mathrm{f}_1 = \bar{\mathrm{f}}_1 \cdot h \quad , \quad \Phi = \bar{\Phi}_1 \cdot h \\
&\mathrm{f}_2 = \bar{\mathrm{f}}_2 \cdot h \quad , \quad \Phi = \bar{\Phi}_2 \cdot h \\
&A_{ij} = \bar{A}_{ij} \cdot E_2 \cdot h \\
&B_{ij} = \bar{B}_{ij} \cdot E_2 \cdot h^2 \\
&D_{ij} = \bar{D}_{ij} \cdot E_2 \cdot h^3
\end{aligned}
\qquad \text{..................44}
$$

After non dimensionalised the equation [43] in matrix form it can be written as in matrix form as given below

$$
[\bar{H}] \cdot \{\bar{X}\} = \bar{\mathrm{v}}^2 \cdot \{\bar{X}\} \qquad \text{........................45}
$$

Where,

$$
\bar{\mathrm{v}}^2 = \frac{\mathrm{w}^2 \mathrm{r}\, h^2}{E_2} \left(\frac{R}{h} \right)^2
$$

$$
[\bar{H}] = [\bar{M}]^{-1} \cdot [\bar{C}]
$$

A non-trivial solution for the column matrix $\{\bar{X}\}$will give the required eigenvalues, which are the values of the square of the frequency parameter $\bar{\mathrm{v}}$ in the present case. The lowest value of $\bar{\mathrm{v}}$ is of particular interest.

CHAPTER - 4

RESULT AND DISCUSSION

CHAPTER- 4

NUMERICAL RESULTS AND DISCUSSIONS

4.1 INTRODUCTION

The theoretical formulation presented in the previous section is a unified analysis for determining the natural frequencies of the forward and backward travelling modes for rotating cylindrical shells by using FSDT. Although the formulation of the problem is general, for reasons of simplicity, results are to be presented only for the cases of simply supported simply supported, four layered [$0^0/90^0/90^0/0^0$], three layered [$0^0/90^0/0^0$] and two layered [$0^0/90^0$] finite length rotating cylindrical shells. Results are also presented, as special cases, for non rotating cylindrical shells.

The frequency parameters are calculated by using a computer program for vibration of laminated orthotropic rotating cylindrical shells. The results obtained using the present theory is compared to other theories and thus the derived formulation is validated. The numerical values of the lowest value of frequency parameter are presented for various shell parameters in this chapter. The frequency envelopes are plotted as a function of L/R (for shells) to study the effect of the number of layers, the thickness of layers and the h/R ratio on frequency.

4.2 SOLUTION OF EIGEN VALUE PROBLEM AND COMPUTER PROGRAM

The equation (43) represents a general Eigen value problem, where ω^2 is the Eigen value and {X} is the eigenvector. For convenience the equation (43) is non-dimensionalised. Then if the equation is pre multiplied by [$\overline{M}$]$^{-1}$, (where the bar indicates the non- dimensionalised form), one obtains the following standard eigenvalue problem,

$$\left[\overline{H}\right]\left\{\overline{X}\right\} = \overline{\omega}^2 \left\{\overline{X}\right\} \quad \text{……………………45}$$

Where,

$$\overline{\omega}^2 = \frac{\omega^2 R^2 \rho h}{A_{22}}$$

$$[\overline{H}] = [\overline{M}]^{-1}[\overline{C}]$$

A non trivial solution for the column matrix $\{\bar{X}\}$ will give the required eigenvalues, which are the values of the square of the frequency parameter $\bar{\omega}$ in the present case. The lowest values of the $\bar{\omega}$ is of particular interest. (for a set of fixed shell parameters, many values of $\bar{\omega}^2$ can be obtained).

A standard subroutine in the computer program to find the eigenvalue of matrices has been used, which consists of root power method of iteration with Wielandt's deflection technique. The program will be called RTPM, which is capable of finding the required number of roots in descending order. The change of the sign of the determinant value is checked for values of one percent on either side of the root to verify the convergence. The RTPM program gives the highest value of eigenvalue first, so if the $[\bar{M}]$ matrix is taken as $[\bar{C}]^{-1}[\bar{H}]$, then the highest value of $\left[\frac{1}{\bar{\omega}^2}\right]$ is obtained, that is the lowest value of $\bar{\omega}^2$ and $\bar{\omega}$ which is of particular interest.

4.3 NUMERICAL RESULTS AND DISCUSSION

4.3.1 The Validation of the Formulation and Numerical Results

Using the formulation developed in the previous sections, numerical studies are carried out. For the validation of the formulation, numerical results with present theory are compared first for non-rotating cylindrical shells as presented by Soldatos [26] in Table 4.1 and Lam and Loy [18] in Table 4.2 and with those of Lam and Loy [18] in Tables 4.3 for rotating cylindrical shells.

The lowest value of the frequencies has been calculated at first for two layers and three layers laminated composite cylindrical shells for various values of m and n. In Table 4.1, comparisons are made with the results of simply supported non-rotating cylindrical shell and are presented. This table shows the present results and those of Soldatos [26] for the non dimensional frequency parameter $\bar{\omega}^2 = \omega^2 L_s^2 \rho h / A_{11}$ of a two-layered [$0^0/90^0$] simply supported simply supported cylindrical shell. The geometrical and material properties used are h/R = 0.01, *L/R* =1.0 and L/R = 2.0 and $E_{11}/E_{22} = 40$, $G_{12}/E_{22} = 0.5$ and $\mu_{12} = 0.25$. It is seen that the present method yields accurate results. From the results presented in this table, it is clear that the present FSDT results are in excellent agreement with those obtained using other methods.

It is well documented that Love developed the first mathematical framework for a thin shell theory which is now known as Love's first approximation theory. Following this, the Kirchhoff-Love hypothesis was put forth and has since become the foundation of many thin shell theories. The thin shell theories developed using the Kirchhoff-Love hypotheses differ from one another when the terms relevant to h/R are retained or neglected in the constitutive and strain-displacement relations.

Table 4.1: Comparison of frequency parameter $\bar{\omega}^2 = \omega^2 L^2 \rho h/E_{22}$ for non-rotating two layered cross ply [$0^0/90^0$] circular cylindrical shell with simply supported boundary condition at both edges (h/R =0.01)

L/R	n	Soldatos [1984]	Present Theory	Error in Percentage
1.0	1	2.106	2.106	0%
	2	1.344	1.3444	0%
	3	0.9589	0.9587	-0.020%
	4	0.7495	0.7493	-0.026%
	5	0.6423	0.6419	-0.062%
2.0	1	1.073	1.073	0%
	2	0.6710	0.6710	0%
	3	0.4710	0.4709	-0.021
	4	0.3774	0.3773	-0.026
	5	0.3632	0.3629	-0.082

Error in Percentage = 100 * (Present Theory - Soldatos)/ soldatos

Table 4.2: Comparison of lowest Nondimensional Frequency Parameter $\overline{v}^{2} = w^{2} L^{2} r \; h/E_{22}$ for a [0°/90°/0°] simply Supported non rotating Laminated Cylindrical Shell (h / R = 0.002)

L/R	n	Lam and Loy [1995]	Present
1	1	1.061284	1.061270
	2	0.804054	0.803997
	3	0.598331	0.598193
	4	0.450144	0.449828
	5	0.345253	0.344547
5	1	0.248635	0.248632
	2	0.107203	0.107185
	3	0.055087	0.054873
	4	0.033790	0.032561
	5	0.025794	0.021375

The free vibration solution for a rotating cylindrical shell is a function of the rotational speed. For a given rotational speed, the two smallest eigen solutions for each mode of the vibration, i.e. for each pair of the wave number (m, n) where m is the axial wave number and n is the circumferential wave number, consist of positive and negative eigen values. These two eigen values correspond to the natural frequencies for the backward and forward travelling waves or to the natural frequencies for clockwise and anticlockwise rotational speed of the shell. The positive eigenvalue corresponding to the backward waves is due to the rotation in clockwise direction (i .e. $\Omega > 0$) and the negative eigenvalue corresponding to the forward waves is due to a rotation in anticlockwise direction (i. e. $\Omega < 0$). In the case of stationary shell, these two eigenvalues are identical and the vibratory motion of the shell is a standing wave motion. However, as the shell starts to rotate, this standing wave motion is transformed and depending on the direction of rotation, backward or forward waves are present. Two rotational effects are introduced when the shell rotates at higher speed; one is centrifugal effect and the other is the Coriolis effect. To neglect these two effects in present study, the speed of rotation is restricted up to 1.0 revolution per second (i.e. Ω = 0.0, 0.1, 0.4, 1.0 revolutions per second).

To further verify the present analysis, the results are compared to those presented by Lam and Loy (1995) for the cross-ply laminated cylindrical shells of lamination scheme [0°/90°/0°]. The comparisons for the non-rotating shell with L/R =1 and L/R = 5 are presented in Table 4.2. The geometrical and material properties used are E_{11} = 19GPa, E_{22} = 7.5GPa and • = 0.26. It is obvious that good agreement is achieved for all cases. Similarly for the rotating cylindrical shells, corresponding comparisons are presented in Tables 4.3 and 4.4 in which $\bar{v}_b$ and $\bar{v}_f$ are the backward-wave and forward-wave non-dimensional frequency parameters, respectively. The material properties and layer configuration are chosen as earlier. Again it is evident that very good agreement is achieved, thus further verifying the validity and accuracy of the present formulation.

Table 4.3: Comparison of lowest Nondimensional Frequency Parameter $\bar{v}^2 = w^2 L^2 r\ h/E_{22}$ for a [0°/90°/0°] simply Supported Rotating Laminated Cylindrical Shell (h / R = 0.002, L/R =1)

Ω (rps)	n	Lam and Loy [1995] Backward	Lam and Loy [1995] Forward	Present Backward	Present forward
0.1	1	1.061429	1.061140	1.061281	1.061257
	2	0.804214	0.803894	0.804012	0.803981
	3	0.598476	0.598157	0.598214	0.598172
	4	0.450270	0.450021	0.449856	0.449800
	5	0.345363	0.345149	0.344584	0.344510
0.4	1	1.061862	1.060706	1.061459	1.061080
	2	0.804696	0.803415	0.804244	0.803749
	3	0.598915	0.597762	0.598528	0.597858
	4	0.450662	0.449667	0.450278	0.449378
	5	0.345724	0.344870	0.345138	0.343955

Table 4.4: Comparison of lowest Nondimensional Frequency Parameter $\bar{\omega}^{2}=\omega^{2}L^{2}\rho h/E_{22}$ for a [0°/90°/0°] simply Supported Rotating Laminated Cylindrical Shell (h/R = 0.002, L/R = 5)

Ω (rps)	n	Lam and Loy [1995]		Present	
		Backward	Forward	Backward	forward
0.1	1	0.248917	0.248352	0.248669	0.248594
	2	0.107436	0.106972	0.107293	0.107076
	3	0.055267	0.054916	0.055096	0.054648
	4	0.033945	0.033669	0.032945	0.032171
	5	0.025943	0.025836	0.021962	0.020770
0.4	1	0.249765	0.247504	0.249239	0.248022
	2	0.108143	0.106288	0.108896	0.105446
	3	0.055868	0.054466	0.058347	0.051163
	4	0.034608	0.033507	0.038249	0.025639
	5	0.026825	0.025924	0.029397	0.007041

4.3.2: Numerical results

For further study of the effects of various shell parameters on rotating cylindrical shells, the material properties used and the different cross-ply lay-ups are shown in Table 4.5.

Table 4.5: Material properties and layer thicknesses for various cross-ply shells

Cylindrical shell	Layer	Thickness (mm)	E (N/m^2)	Poisson's Ratio (m)
Two Layer	0^0	h/2	19.0 x 10^9	0.26
	90^0	h/2	7.5 x 10^9	0.26
Three layer	0^0	h/3	19.0 x 10^9	0.26
	90^0	h/3	7.5 x 10^9	0.26
	0^0	h/3	19.0 x 10^9	0.26
Four Layer	0^0	h/4	19.0 x 10^9	0.26
	90^0	h/4	7.5 x 10^9	0.26
	90^0	h/4	7.5 x 10^9	0.26
	0^0	h/4	19.0 x 10^9	0.26

4.3.3 Influence of rotating velocity

The effects of rotating angular velocity in rotating shells constitute one of the most critical investigations of this monograph. Physically, the important differences between the rotating and non rotating shells of revolution are the Coriolis and centrifugal acceleration, as well as the hoop tension arising in rotating shells due to the angular velocities. These effects have significant influence on the dynamic behavior of the rotating shells. For example, the frequency characteristics of a stationary shell structure are generally determined by the shell geometry, material properties and boundary condition. However when the same shell rotates, the structural frequency characteristics are qualitatively altered. This qualitative difference manifests itself in the form of bifurcation phenomena in the natural frequency parameters. For stationary shell revolution, the vibration of the shell is a standing wave motion. However, when the same shell rotates, the standing wave motion is transformed, and depending on the direction of rotation, backward or forward waves will emerge. In this section, the discussion is made for the influence of the rotating velocity on the frequency characteristics of the rotating circular cylindrical shell.

4.3.3.1 Influence due to the circumferential wave number and L / R

Table 4.6, Table 4.7 and Table 4.8 shows the variation of the natural frequency ω with circumferential wave number n for the two, three and four layer cross-ply lay-ups respectively. The Table 4.7 illustrates the effect of rotation on the frequency of the rotating multi layered cylindrical shells which has three layers of construction, where the thickness of middle layer (E_{22} =7.6 G N/ m^2, μ = 0.26, and ρ = 1643 Kg/m^3) is the same as that of the outer layers (E_{11} =19.0 G N/ m^2, μ = 0.26, and ρ = 1643 Kg/m^3). The Tables show the variation of the frequency parameter with the circumferential wave number 'n' for various speeds of rotation and L/R ratios (L/R = 1, 5, 10), considered for both the stationary as well as rotating cylindrical shells. When an angular velocity is introduced to the shell, there is an increase in the frequency parameter. It is observed that the natural frequency characteristics of rotating shells are however not very different from those of stationary shell for these speeds of rotation. The frequency parameter $\bar{v}$ decreases rapidly for small circumferential wave number $(n \le 3)$. It is seen that the increasing value of rotating speed Ω increases the frequency parameter and with the increase in the circumferential wave number n there is a decrease in the frequency parameter $\bar{v}$ for a particular L/R ratio.

Table 4.6 Frequency parameter for the two layered cross ply cylindrical shell [0^0/90^0].

Ω (rps)	L/R=1	n =1	n = 2	n = 3	n = 4	n = 5	n = 6
0	1	0.820836	0.598944	0.435591	0.322881	0.245026	0.190317
	5	0.175633	0.074974	0.038286	0.022698	0.014895	0.010487
	10	0.058649	0.020886	0.010081	0.005847	0.003796	0.002657
0.1	1	0.820844	0.598955	0.435607	0.322903	0.245055	0.190354
	5	0.175663	0.075059	0.038464	0.023004	0.015363	0.011145
	10	0.058738	0.021193	0.010739	0.006942	0.005345	0.004616
0.4	1	0.820970	0.599127	0.435846	0.323228	0.245487	0.190914
	5	0.176106	0.076331	0.041042	0.027187	0.021173	0.018380
	10	0.060063	0.025360	0.017913	0.016073	0.015523	0.015328

Table 4.7 Frequency parameter for the three layered symmetric cross ply cylindrical shell [$0^0/90^0/0^0$].

Ω (rps)	L/R	n =1	n =2	n =3	n = 4	n =5	n = 6
0	1	0.85293	0.659553	0.498287	0.378423	0.291799	0.291094
	5	0.207618	0.092064	0.047344	0.028141	0.018489	0.013026
	10	0.07198	0.025895	0.012522	0.007267	0.00472	0.003305
0.1	1	0.852941	0.659566	0.498304	0.378447	0.29183	0.229149
	5	0.20765	0.092153	0.047527	0.028456	0.01897	0.013704
	10	0.072073	0.026215	0.0132	0.008424	0.006348	0.005389
0.4	1	0.853091	0.659764	0.498571	0.378801	0.292292	0.229741
	5	0.208119	0.093473	0.050196	0.032815	0.025099	0.021437
	10	0.07345	0.03056	0.02087	0.018384	0.017621	0.017345

The Table 4.8 illustrates the effect of rotation on the frequency of multi layered cylindrical shell which has four layers of construction [i.e. $0^0/90^0/90^0/0^0$], where the thickness of middle two layers (E_{22} = 7.6 G N/ m^2, m =0.26, and r = 1643 Kg/m^3) is equal to that of the outer two layers (E_{11} =19.0 G N/ m^2, m =0.26, and r = 1643 Kg/m^3). The Table shows the variation of the length to radius ratio (L/R = 1, 5, 10) for both the stationary as well as rotating shell. It is observed that the general frequency characteristics of rotating shells are slightly higher than those of stationary shell. The frequency parameter $\bar{v}$ decreases rapidly for small circumferential wave number $(n \leq 3)$. It is again seen that with the increasing value of rotating speed Ω, there is an increase in the frequency parameter. With the increasing circumferential wave number n, there is a decrease in the frequency parameter $\bar{v}$. As the circumferential wave number i.e. n is increasing beyond 3 (i.e. n >3) there is not much variation in the frequency parameter for L/R ratio 10.

Table 4.8 Frequency parameter for the four layered symmetric cross ply cylindrical shell [$0^0/90^0/90^0/0^0$]

Ω (rps)	L/R	n =1	n = 2	n = 3	n = 4	n = 5	n = 6
0	1	0.8329	0.613571	0.44746	0.331516	0.251093	0.194553
	5	0.179935	0.075999	0.038599	0.022826	0.01496	0.010525
	10	0.059463	0.021005	0.010118	0.005863	0.003806	0.002663
0.1	1	0.832908	0.613582	0.447475	0.331537	0.251121	0.194589
	5	0.179964	0.076084	0.038775	0.023131	0.015425	0.011181
	10	0.059551	0.02131	0.010773	0.006956	0.005362	0.004619
0.4	1	0.833033	0.61375	0.447708	0.331854	0.251543	0.195137
	5	0.180399	0.077339	0.041334	0.027294	0.021219	0.018402
	10	0.060859	0.025459	0.017933	0.016079	0.015526	0.01533

4.3.4 Influence of length and thickness

There are generally many physical and geometrical parameters which influence the frequency characteristics of rotating shells. Physical parameters include the rotating angular velocity, material properties and boundary conditions. The major geometrical parameters include the length (L), radius (R), and thickness (h). In this section, discussions are made on the influence of the geometrical length ratio L/R and thickness ratio h/R on the frequency characteristics of the rotating multi layered circular cylindrical shells. The cylindrical shell has three layers of construction, where the thickness of the middle layer (E = 7.6 GPa, m = 0.26, and r = 1634 Kg/m^3) is equal to that of the two surface layers (E = 19.0 GPa, m = 0.26, and r = 1634 Kg/m^3).

Figure 4.1 shows the variation of the natural frequency w (Hz) with the rotating velocity at various thickness-to-radius ratios (h/R) and constant length-to-radius (i.e. L /R =10) for the rotating multilayered cylindrical shell with the simply supported boundary condition at the edges. It is observed that for very thin shell (i.e. h/R = 0.002) the natural frequency parameter rapidly increases as the rotating velocity of the shell increases. But as the thickness of the shell is increases the natural frequency parameter slightly increases for the different thickness-to-radius ratios (i.e. for the case of h /R = 0.002, 0.01, 0.02,). For understanding the behavior of the length parameter simultaneously for different thickness of cylindrical shell Figure 4.2 and figure 4.3 represent the graph for the L/R = 5 and L/R = 1. From these graph it can be seen that as the length increases the frequency is decreasing. It is due to that the as length increases the stiffness of the shell increases and the as stiffness increases it is decreasing the frequency parameters of the shell.

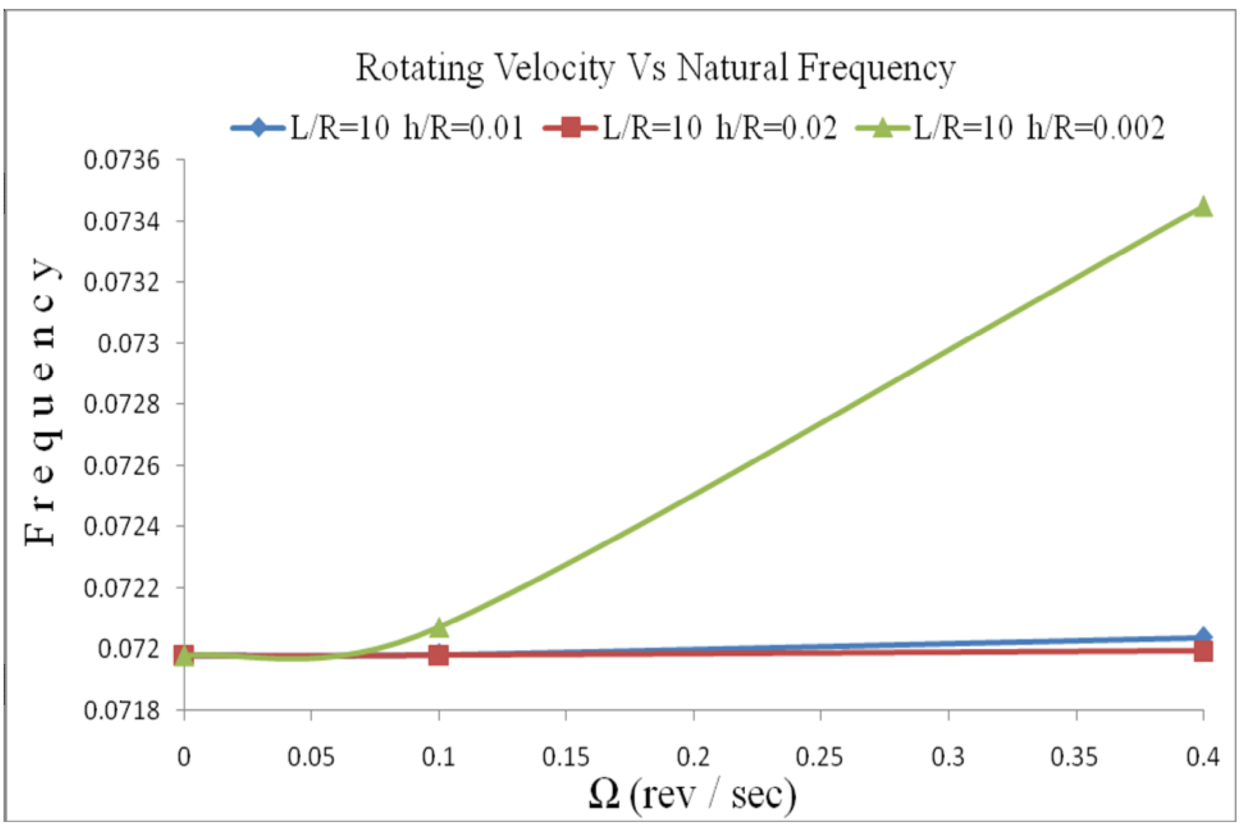


Figure.4.1: Variation of frequency parameter w with thickness-to-radius ratio h / R for three layers symmetric cross-ply laminated shell, m =1 and n =1 and L /R = 10

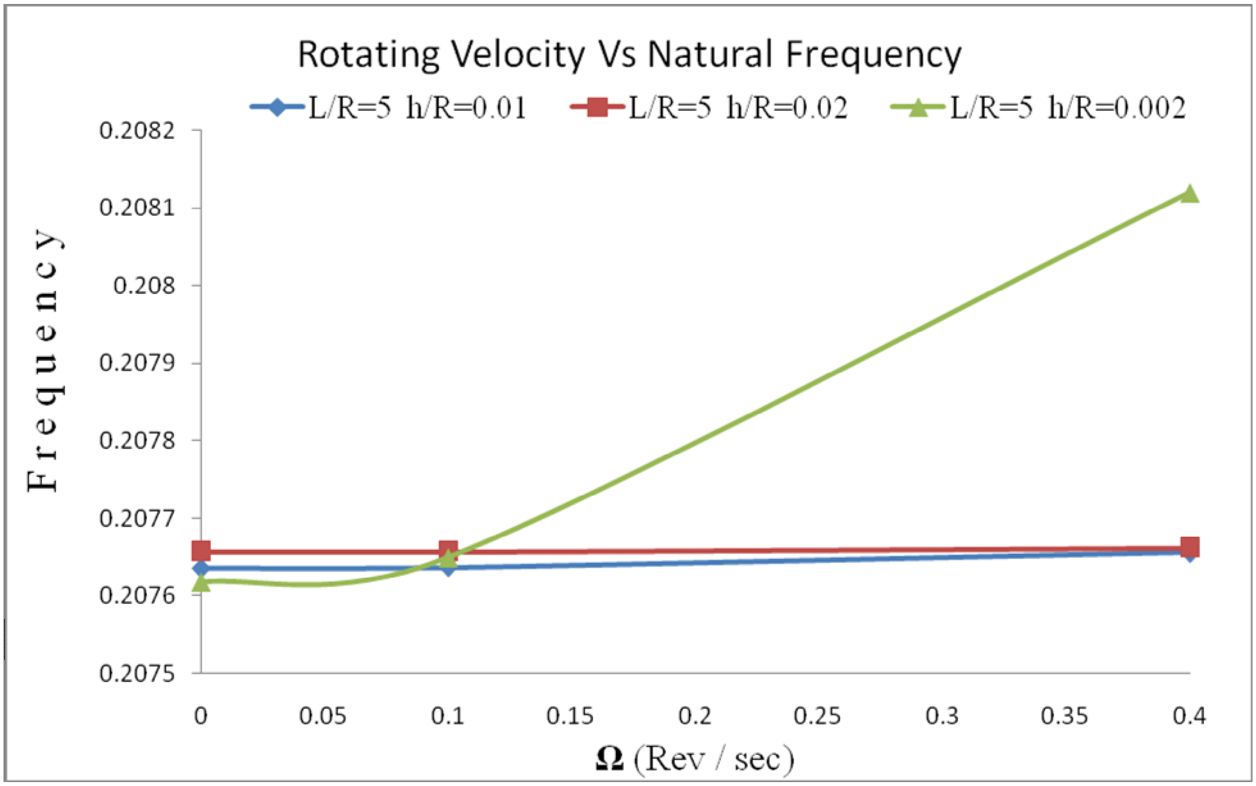


Fig.4.2: Variation of frequency parameter w with thickness-to-radius ratio h / R for three layers symmetric cross-ply laminated shell, m =1 and n =1 and L /R = 5

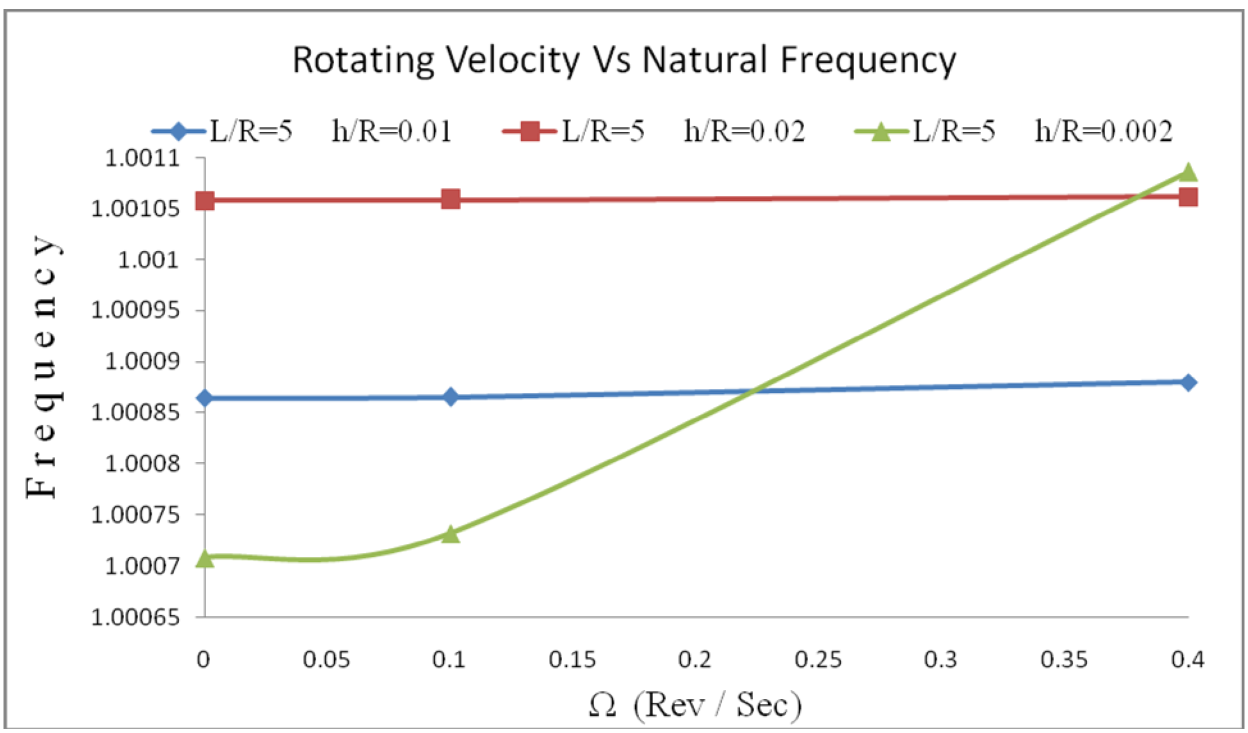


Fig.4.3: Variation of frequency parameter w with thickness-to-radius ratio h / R for three layers symmetric cross-ply laminated shell, m =1 and n =1 and L /R = 5

Figure 4.4 shows the variation of the natural frequencies w (Hz) with the length-to-radius ratios (L / R) for different rotating velocity Ω (rps) and for constant h/R = 0.002 for the rotating three layered cylindrical shell (0°/ 90°/ 0°) with the simply supported boundary condition at both edges. It is observed that natural frequency decreases with the length-to thickness ratio (L / R) for the rotational velocities Ω. But the overlapping of the graphs illustrate that the there is no variation due to the rotation. It is further observed that the influence of the length ratio L/R on the natural frequency of the rotating cylindrical shell is larger than of the thickness ratio h/R.

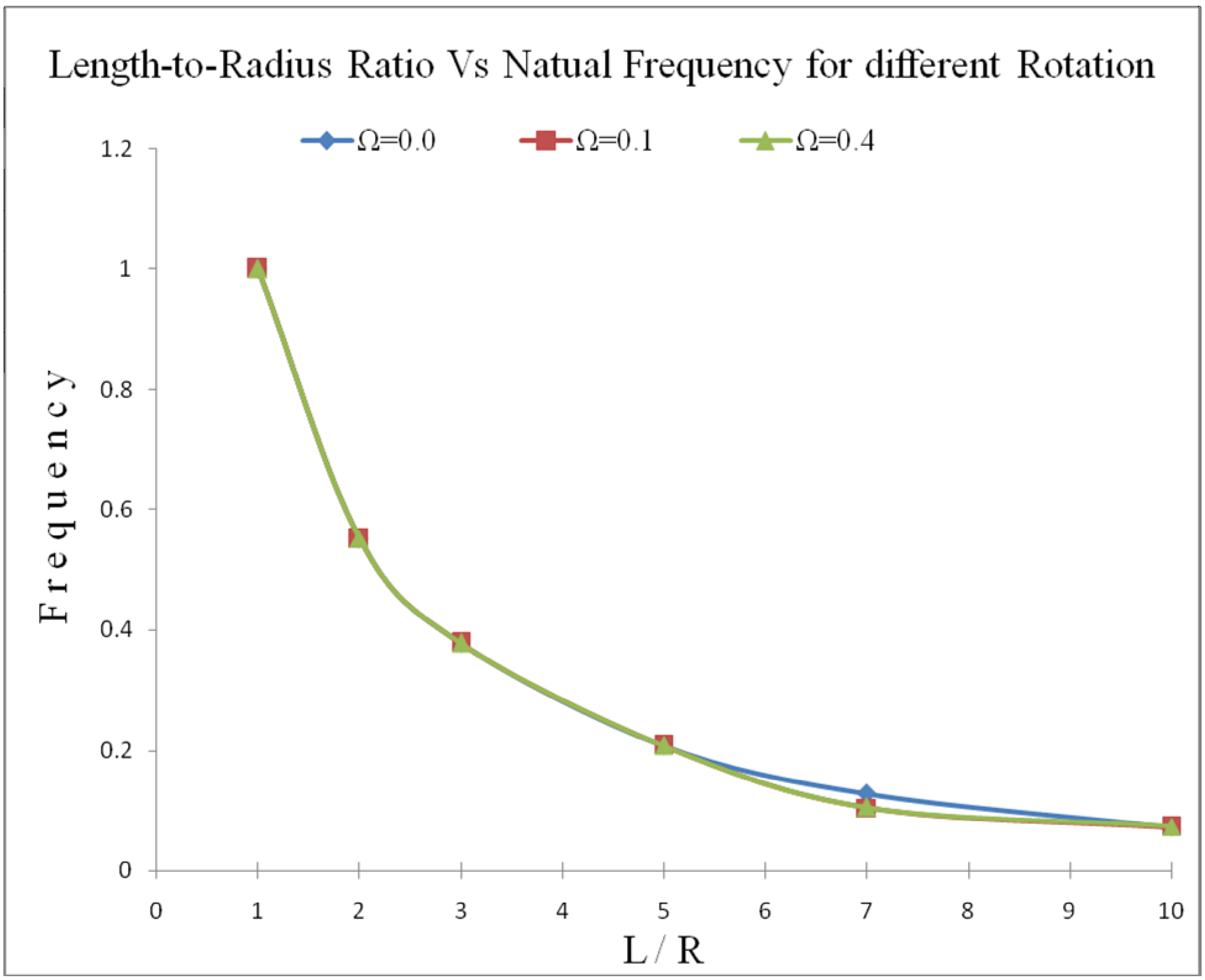


Figure 4.4: Variation of the natural frequencies w (Hz) with length-to-radius ratios (L/R) for different rotating velocity Ω (rps) for three layers cross ply laminated shell with m =1, n =1 and h/R = 0.002

4.3.5 Influence of layer configuration of composites

When the rotating shell is made of composite material, the influence of the layer configuration should be considered since it is one of the most important characteristics of a composite material. Usually layers are made of different isotropic materials, and their principal directions may also be oriented differently. For laminated composites, the fiber directions determine layer orientation. In this section, however, the discussion is made for simplified case where the rotating cylindrical shells are composed of two, three, and four layer cross-ply laminated composite layers. The material properties and layer thicknesses are as shown in Table 4.5.

4.3.5.1 Influence circumferential wave number on different Layer Configuration

Figure 4.3 shows the influence of the layer configuration on the natural frequencies of the rotating shells with simply supported boundary condition at both edges. Figure 4.3 (A) shows the variation of frequency with the circumferential wave number n for non-rotating shells for the different layer configuration. The natural frequency corresponding to the backward waves for all the cylindrical shells decreases with the increase of the circumferential wave number. It is also observed that the three layer configuration (i.e. $0^0/90^0/0^0$) has the highest frequency parameter, followed by the four layers and the natural frequency of the two layer shell is relatively lower. This is true for both rotating and non rotating shells.

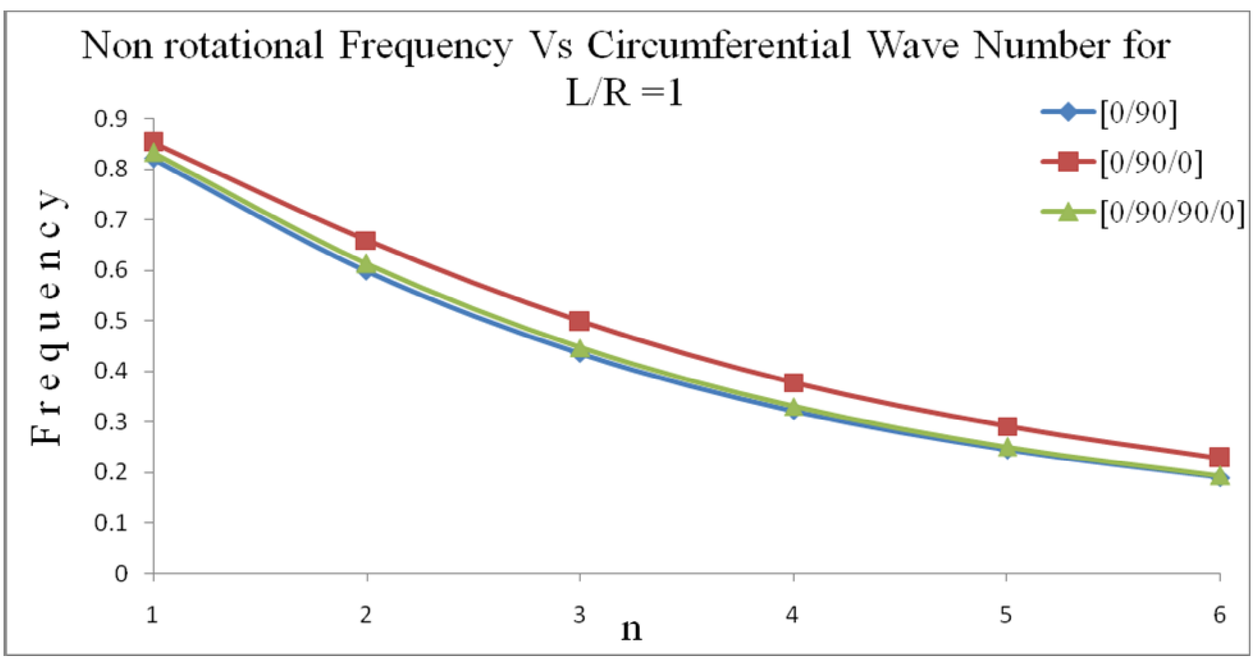
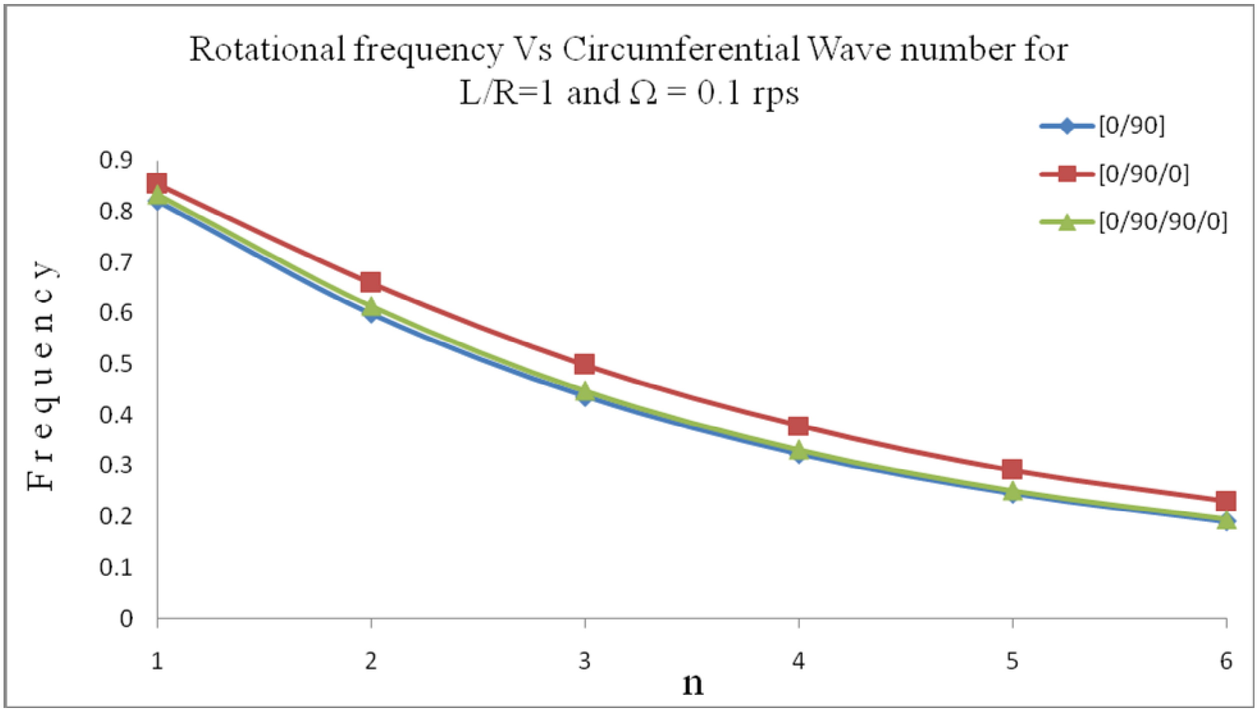


Figure 4.5 (A): Non Rotating cylindrical shell i. e. Ω = 0.0 rev/sec.

B. Rotating cylindrical shell i. e. Ω = 0.1rev/sec.



C. Rotation of the cylindrical shell i. e. Ω = 0.4rev/sec.

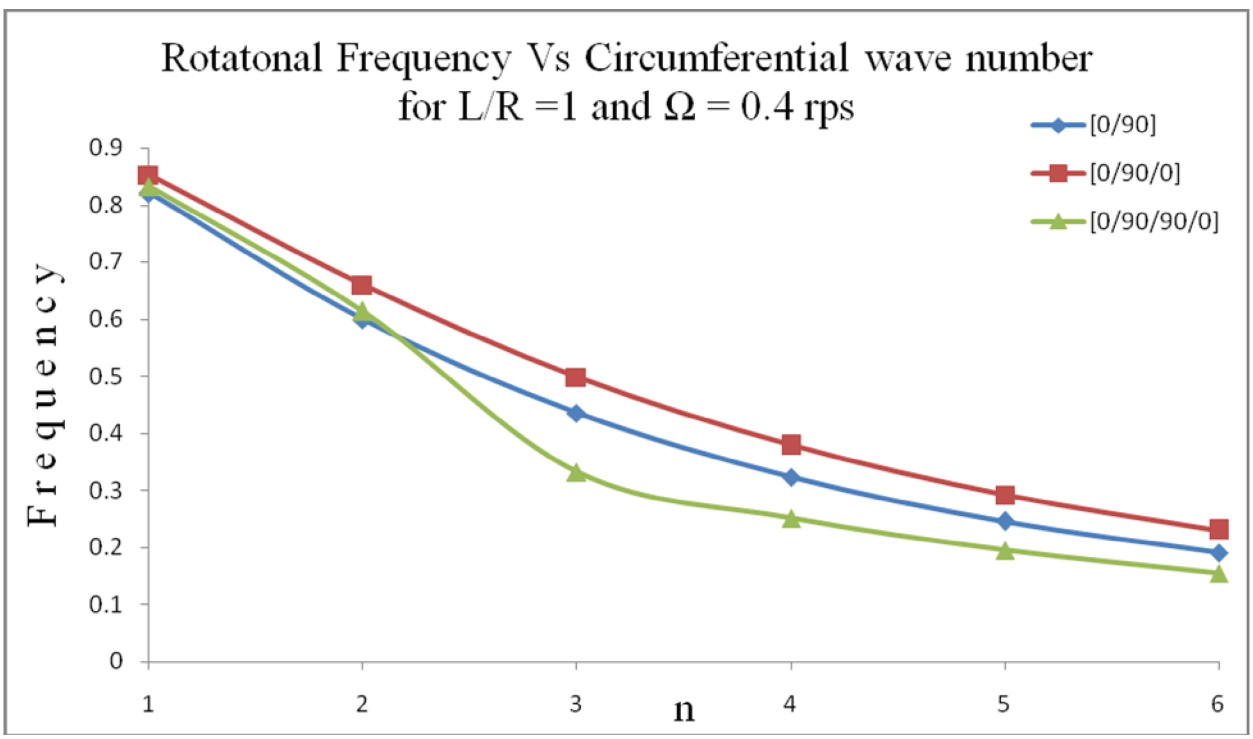


Figure 4.5: Natural frequency as a function of n for simply supported rotating cylindrical shell with different layer configuration (m =1, L/R = 1 , h/R = 0.002) (A) $\Omega = 0$ rps, (B) $\Omega = 0.1$rps, (C) $\Omega = 0.4$ rps.

4.3.5.2 Influence rotating Velocity on different Layer Configuration

Figure 4.4 shows the variation of the natural frequency with the rotating velocity for three different layer configurations (i.e. [0/90], [0/90/0], [0/90/90/0]). It is observed that three layer configuration has the highest natural frequency, followed by the four and the natural frequency for two layers are relatively lower. It is also observed that the as the rotational speed is increased there is very small increment in the natural frequency.

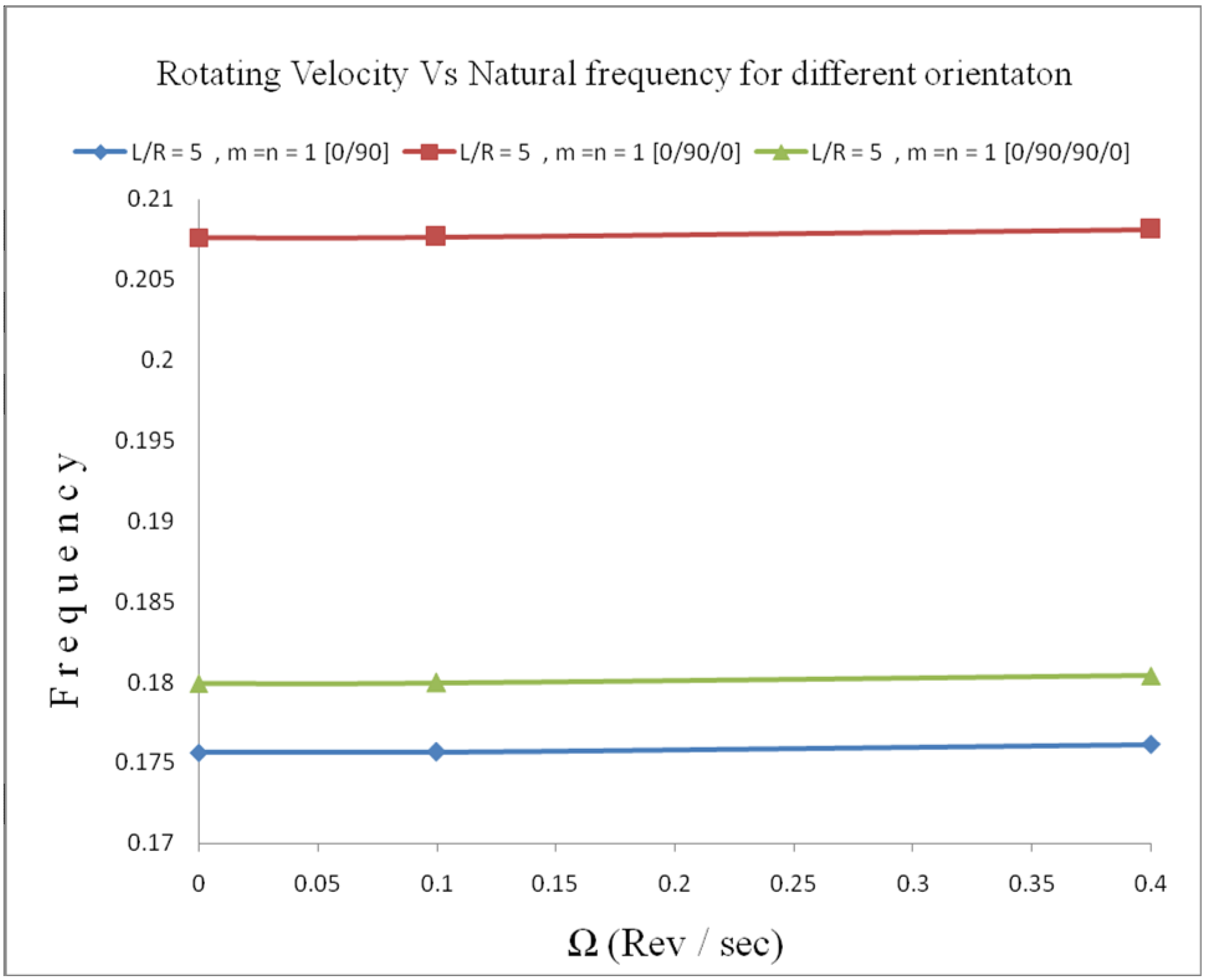


Figure 4.6: Natural Frequency as a function of rotating velocity for simply supported rotating cylindrical shells with different layer configuration (m=1, n=1, h/R= 0.002, L/R =5)

4.3.5.2 Influence of Length-to-radius for different layer configuration

Figure 4.5 shows the variation of the natural frequency with the length-to-radius ratio (L/R) for various layer configurations. As the length ratio L / R is increased for the all the configuration, the natural frequency decreases rapidly and this is subsequently followed by more gradual decrease. Once again three layer configurations have highest natural frequency, followed by the four and two layer lay-ups respectively.

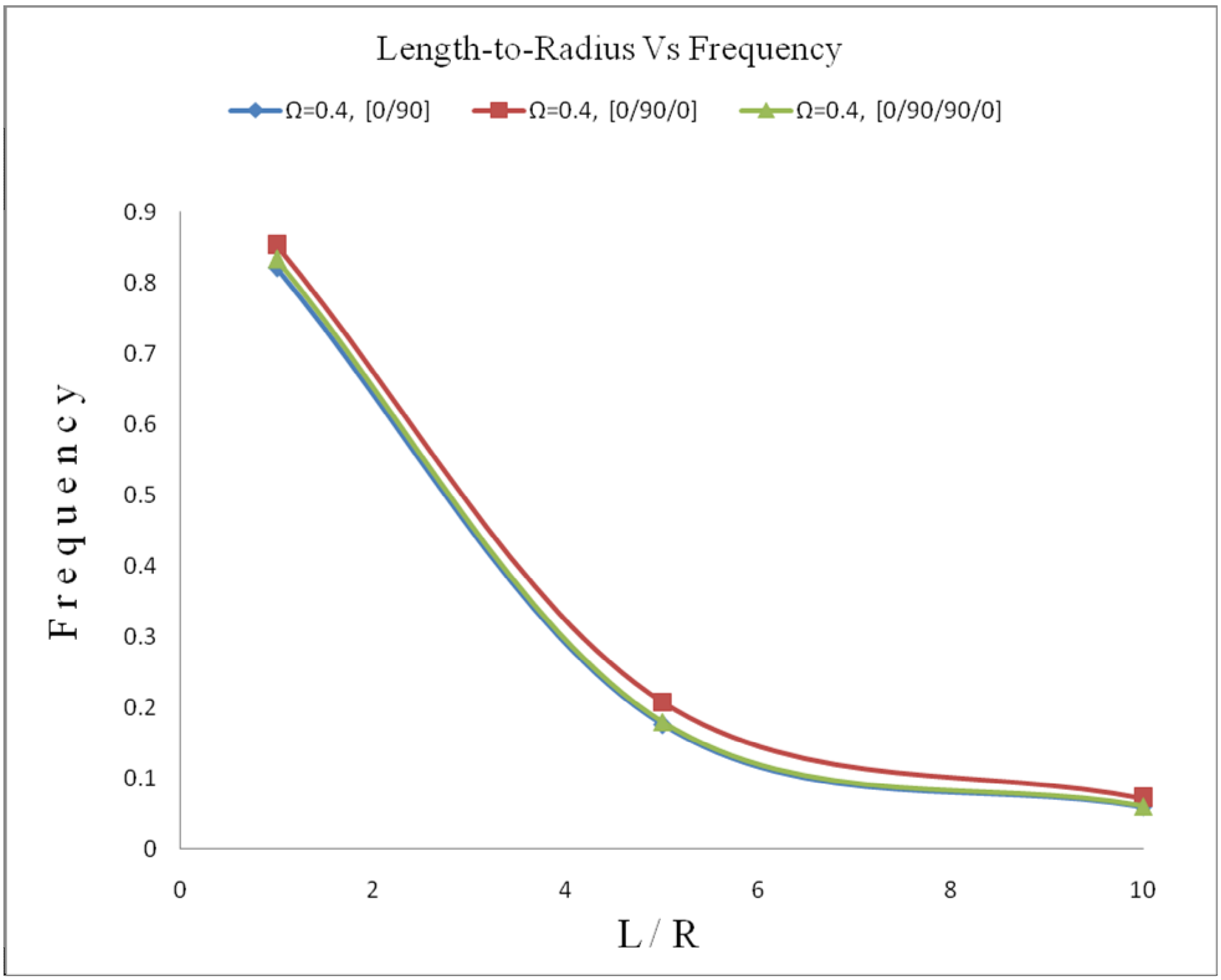


Figure 4.7: Natural frequency as a function of the length-to-radius ratio (L / R) for simply supported rotating cylindrical shells with different layer configuration (Ω = 0.4rps, h / R =0.002)

CHAPTER - 5

CONCLUSION

CHAPTER-5

CONCLUSION

The free vibration problem of laminated thin rotating cylindrical shells is analyzed in this study by first order shear deformation theory, a versatile classical procedure. The illustrative examples of cylindrical shells with simply supported boundary conditions for convenience are considered. The analysis is quite capable of dealing with the vibration problems of composite cylindrical shells with arbitrary end conditions. Versatility and validity of first order shear deformation theory is also aptly illustrated by comparing the results from the present work with the corresponding results in previous studies using quite different alternative approaches. It is observed that there is a very good agreement between various sets of results.

The effects of the rotating velocity, circumferential wave number, length-to-radius ratio, thickness-to-radius ratio, composite lamination, and geometrical properties were investigated on the vibration characteristics of a thin rotating cylindrical shell. From the analysis of the result presented in previous section the following concluding remarks can be drawn:

1. The frequency parameter increases with angular velocity of rotation, though for the small speeds considered the increase is not much in comparison with the non-rotating shells.
2. The natural frequencies rapidly decrease with circumferential mode number ‘n’ up to mode number of 3 and for mode number beyond 3 the natural frequency gets near about constant for L/R of 10.
3. The rotational motion has a significant influence on the natural frequencies and these influences depend largely on the velocity of rotation.
4. The influence of length-to-radius ratio on natural frequency of the rotating cylindrical shell is larger than that of thickness-to-radius ratio.
5. For the two layers configuration, the natural frequency parameter is lowest, while it is highest for the three layers as compared to the four layers configuration for a particular angular velocity of rotation and particular shell geometrical parameters.

FUTURE WORK

The method studied in the present study can be used to evaluate the vibration characteristics for the higher speed of rotation and thick rotating cylindrical shell. However, this study is limited to the lower rotating velocity and thin rotating cylindrical shell. This is due to the neglecting the Coriolis and centrifugal effect. So considering the Coriolis and centrifugal effect the present study can be used for higher rotating velocity and thick shells too. The present study can be extended for the angle ply laminated shell.

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Appendix

APPENDIX

The coefficients of the [C_{ij}] and [M_{ij}] matrices are given below.

$$C(1,1) = A_{11}\lambda_m^2 + A_{66}n^2$$

$$C(1,2) = A_{12}\lambda_m n + A_{66}\lambda_m n + \frac{B_{12}}{R}\lambda_m n + \frac{B_{66}}{R}\lambda_m n$$

$$C(1,3) = -A_{12}\lambda_m$$

$$C(1,4) = B_{11}\lambda_m^2 + B_{66}n^2$$

$$C(1,5) = B_{12}\lambda_m n + B_{66}\lambda_m n$$

$$C(2,1) = C(1,2)$$

$$C(2,2) = A_{66}\lambda_m^2 + A_{22}n^2 + 2\frac{B_{66}}{R}\lambda_m^2 + 2\frac{B_{22}}{R}n^2 + \frac{D_{66}}{R^2}\lambda_m^2 + \frac{D_{22}}{R^2}n^2 + \rho h \cdot R^2 \cdot \Omega^2$$

$$C(2,3) = -A_{22}n - \frac{B_{22}}{R}n$$

$$C(2,4) = B_{12}\lambda_m n + B_{66}\lambda_m n + \frac{D_{66}}{R}\lambda_m n + \frac{D_{21}}{R}\lambda_m n$$

$$C(2,5) = B_{66}\lambda_m^2 + \frac{D_{66}}{R}\lambda_m^2 + B_{22}n^2 + \frac{D_{22}}{R}n^2$$

$$C(3,1) = C(1,3)$$

$$C(3,2) = C(2,3)$$

$$C(3,3) = A_{55}\lambda_m^2 + A_{44}n^2 - A_{22} + \rho h \cdot R^2 \cdot \Omega^2$$

$$C(3,4) = A_{55}\lambda_m R - B_{12}\lambda_m$$

$$C(3,5) = A_{44}n - B_{22}n$$

$$C(4,1) = C(1,4)$$

$$C(4,2) = C(2,4)$$

$$C(4,1) = C(1,4)$$

$$C(4,2) = C(2,4)$$

$$C(4,3) = C(3,4)$$

$$C(4,4) = D_{11}\lambda_m^2 + D_{66}n^2 + A_{55}R^2$$

$$C(4,5) = D_{12}\lambda_m n + D_{66}\lambda_m n$$

$$C(5,1) = C(1,5)$$

$$C(5,2) = C(2,5)$$

$$C(5,3) = C(3,5)$$

$$C(5,4) = C(4,5)$$

$$C(5,5) = D_{66}\lambda_m^2 + D_{22}n^2 + A_{44}R^2$$

$$M(1,1) = \rho \cdot h \cdot R^2 \quad , \quad M(2,2) = \rho \cdot h \cdot R^2 \quad , \quad M(3,3) = \rho \cdot h \cdot R^2$$

$$M(4,4) = \frac{\rho \cdot h^3}{12} \cdot R^2 \qquad , \qquad M(5,5) = \frac{\rho \cdot h^3}{12} \cdot R^2$$

The elements of the [M] matrix, which are not given above are zero.

The non-dimensionalised coefficients of the $\left[\overline{C}\right]$ and $\left[\overline{M}\right]$ matrices are given below.

$$\overline{C}(1,1)=\overline{A}_{11}\lambda_m^2+\overline{A}_{66}n^2$$

$$\overline{C}(1,2)=\overline{A}_{12}\lambda_m n+\overline{A}_{66}\lambda_m n+\frac{\overline{B}_{12}}{R}\lambda_m n+\frac{\overline{B}_{66}}{R}\lambda_m n$$

$$\overline{C}(1,3)=-\overline{A}_{12}\lambda_m$$

$$\overline{C}(1,4)=\overline{B}_{11}\lambda_m^2+\overline{B}_{66}n^2$$

$$\overline{C}(1,5)=\overline{B}_{12}\lambda_m n+\overline{B}_{66}\lambda_m n$$

$$\overline{C}(2,1)=\overline{C}(1,2)$$

$$\overline{C}(2\;2)=\overline{A}_{66}\lambda_m^2+\overline{A}_{22}n^2+2\frac{\overline{B}_{66}}{R}\lambda_m^2+2\frac{\overline{B}_{22}}{R}n^2+\frac{\overline{B}_{66}}{R^2}\lambda_m^2+\frac{\overline{D}_{22}}{R^2}n^2+\frac{\rho\cdot h^2\cdot}{E_2}\left(\frac{R}{h}\right)^2\cdot\Omega^2$$

$$\overline{C}(2,3)=-\overline{A}_{22}n-\frac{\overline{B}_{22}}{R}n$$

$$\overline{C}(2,4)=\overline{B}_{12}\lambda_m n+\overline{B}_{66}\lambda_m n+\frac{\overline{D}_{66}}{R}\lambda_m n+\frac{\overline{D}_{21}}{R}\lambda_m n$$

$$\overline{C}(2,5)=\overline{B}_{66}\lambda_m^2+\frac{\overline{D}_{66}}{R}\lambda_m^2+\overline{B}_{22}n^2+\frac{\overline{D}_{22}}{R}n^2$$

$$\overline{C}(3,1)=\overline{C}(1,3)$$

$$\overline{C}(3,2)=\overline{C}(2,3)$$

$$\overline{C}(3,3)=\overline{A}_{55}\lambda_m^2+\overline{A}_{44}n^2-\overline{A}_{22}+\frac{\rho\cdot h^2}{E_2}\cdot\left(\frac{R}{h}\right)^2\cdot\Omega^2$$

$$\overline{C}(3,4)=\overline{A}_{55}\lambda_m R-\overline{B}_{12}\lambda_m$$

$$\overline{C}(3,5)=\overline{A}_{44}n-\overline{B}_{22}n$$

$$\overline{C}(4,1)=\overline{C}(1,4)$$

$$\overline{C}\,(4,2) = \overline{C}\,(2,4)$$

$$\overline{C}\,(4,1) = \overline{C}\,(1,4)$$

$$\overline{C}\,(4,2) = \overline{C}\,(2,4)$$

$$\overline{C}\,(4,3) = \overline{C}\,(3,4)$$

$$\overline{C}\,(4,4) = \overline{D}_{11}\lambda_m^2 + \overline{D}_{66}n^2 + \overline{A}_{55}R^2$$

$$\overline{C}\,(4,5) = \overline{D}_{12}\lambda_m n + \overline{D}_{66}\lambda_m n$$

$$\overline{C}\,(5,1) = \overline{C}\,(1,5)$$

$$\overline{C}\,(5,2) = \overline{C}\,(2,5)$$

$$\overline{C}\,(5,3) = \overline{C}\,(3,5)$$

$$\overline{C}\,(5,4) = \overline{C}\,(4,5)$$

$$\overline{C}\,(5,5) = \overline{D}_{66}\lambda_m^2 + \overline{D}_{22}n^2 + \overline{A}_{44}R^2$$

$$\overline{M}\,(1,1) = \frac{\rho\cdot h^2}{E_2}\cdot\left(\frac{R}{h}\right)^2 \quad , \quad \overline{M}\,(2,2) = \frac{\rho\cdot h^2}{E_2}\cdot\left(\frac{R}{h}\right)^2 \quad , \quad \overline{M}\,(3,3) = \frac{\rho\cdot h^2}{E_2}\cdot\left(\frac{R}{h}\right)^2$$

$$\overline{M}\,(4,4) = \frac{\rho\cdot h^2}{12E_2}\cdot\left(\frac{R}{h}\right)^2 \quad , \quad \overline{M}\,(5,5) = \frac{\rho\cdot h^2}{12E_2}\cdot\left(\frac{R}{h}\right)^2$$

The elements of the $\overline{M}$ matrices, which are not given above, are zero.